# **DETECTION AND ATTRIBUTION**

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First, an illustration of a climate-data application of principal components regression (PCR)

We have proxy data from tree rings for the period 1400–1980 (70 tree rings in total, only first 14 smoothed series shown in plot)

We also have observational data on global temperature anomalies since about 1850 (we only actually use the data 1902–1980).

Question: how best to combine the two, to reconstruct global temperatures back to 1400?

Tree ring data (first 14 only)





Year

We could regress the observed temperatures 1902–1980 on the 70 proxies, then use the fitted regression coefficients to reconstruct temperatures backwards in time.

However, a regression including 70 variables based on 79 observations is doubtful....

### **Proposed New Method**

- 1. Calculate principal components analysis (PCA) of proxies.
- 2. For given  $K \ge 1$ , regress true global temperature anomaly  $y_t$  on first K PCs for 1902–1980:

$$y_t = \beta_0 + \sum_{k=1}^K \beta_k u_{kt} + \epsilon_t$$

3. Form reconstructed  $\hat{y}_t$  for 1400–1980:

$$\hat{y}_t = \hat{\beta}_0 + \sum_{k=1}^K \hat{\beta}_k u_{kt}$$

- 4. Also compute weighted 25-year tapered MAs from  $\hat{y}_t$ , and 90% prediction intervals by the standard method used to compute prediction intervals from linear regression
- 5. Repeated for K = 1, 2, 5, 10, 20, 70.











K=50



Six reconstructions of historical temperature anomalies with pointwise 90% prediction intervals



Reconstruction K=5 with directly measured temperatures



The original "hockey stick curve" (IPCC, 2001), based on Mann-Bradley-Hughes (1998, 1999)



First PC recalculated, scaled and centered to 1902–1980



Second PC recalculated, scaled and centered to 1902–1980

### **Summary**

- 1. Taking just the first PC (K = 1) seems useless reconstruction has very small variance
- 2. However, even K = 2 is much better and shows a clear "hockey stick" shape
- 3. Results similar for K = 2, 5, 10. For K = 20, 50 there is evidence that the regression is overfitting in 1902–1980
- 4. Analysis doesn't (yet) account for autocorrelation
- 5. For a more comprehensive analysis of a different dataset, see Li, Nychka and Ammann (*Tellus* **59A**, 591–598, 2007)

# **Detection and Attribution**

Basic idea: How can we measure the agreement between an artificially generated climate signal from a climate model, and real data as measured by surface observation stations or satellites?

Both observations and model data aggregated into both the observational and model-generated data are aggregated into grid cells, but several thousand of them.

Example: Use 5<sup>o</sup> latitude and longitude grid cells, implying 36 latitude classifications (from 90<sup>o</sup>N to 90<sup>o</sup>S), and similarly 72 longitude increments, giving a total of 2,592 grid cells.

Use  $\ell$  to denote the number of grid cells. One common kind of study is to generate m possible "response patterns"  $x_k$ , k =1, ..., m using the climate model, each one an  $\ell$ -dimensional time series corresponding to the response to one particular kind of forcing factor. For example,  $x_1$  may be the response due to the increase in carbon dioxide,  $x_2$  the response due to changes in sulfate aerosols,  $x_3$  the response due to change in solar forcing,  $x_4$  the response due to volcanic eruptions. Assuming that the observed climate signal is a linear combination of the components due to different forcing factors, this suggests a model of the form

$$y = X\beta + u,$$

where y is  $\ell \times 1$ ,  $X = (x_1 \dots x_m)$  is a  $\ell \times m$  matrix consisting of all the modeled responses to forcing factors, and  $\beta$  is the vector of coefficients which we are trying to estimate.

Here  $\beta_1$  could be the main component of interest (the greenhouse gas effect), while the full family of regression coefficients  $\beta_1, \beta_2, ...$  is needed to determine the influence of all the components.

We say that the greenhouse gas signal is *detected* if we can calculate an estimate and standard error for  $\beta_1$ , that lead to rejection of the null hypothesis  $H_0$ :  $\beta_1 = 0$ .

Given that we have detected a signal, we then estimate all the coefficients  $\beta_1, \beta_2, ...$  to *attribute* the observed climate change to the different components.

If the residual vector u has covariance matrix  $C_N$ , then the optimal generalized least squares (GLS) estimator of  $\beta$  is

$$\hat{\beta} = (X^T C_N^{-1} X)^{-1} X^T C_N^{-1} y, \qquad (1)$$

and

$$\operatorname{Var}\{\widehat{\beta}\} = (X^T C_N^{-1} X)^{-1}.$$
 (2)

The crucial difficulty is that  $C_N$  is unknown and therefore we need some reliable method of estimating it.

One way of deriving (1) and (2) from the standard ordinary least squares (OLS) regression equations is to define a matrix P so that  $PC_NP^T = I_\ell$ , and then to write the regression equation in the form  $Py = PX\beta + Pu$  where the covariance matrix of Pu is  $I_\ell$ . The OLS estimator for  $\beta$  is then  $\hat{\beta} = (X^T P^T P X)^{-1} X^T P^T P y$  with covariance  $(X^T P^T P X)^{-1}$ . But if P and  $C_N$  are invertible we will have  $P^T P = C_N^{-1}$ , which leads to (1) and (2).

Usual approach in climate studies: take model observations from runs of the GCM in stationary conditions without forcing factors, typically 1,000–2,000 years long. With observations  $Y_N$  from N years' of unforced model runs centered to 0 mean, estimate

$$\widehat{C}_N = \frac{1}{n} Y_N Y_N^T. \tag{3}$$

Difficulty with (3): typically  $\ell > n$  and so  $\hat{C}_N$  is a singular matrix. Even though this in itself is not a fatal objection, e.g. valid versions of (1) and (2) can be given involving generalized inverses, the real difficulty is that with so many degrees of freedom in the covariance function, the low-amplitude components of covariance are not reliably estimated, and therefore, any direct attempt to apply (2) by substituting  $\hat{C}_N$  for  $C_N$  will give poor estimates for  $\hat{\beta}$  and innaccurate estimates of uncertainty.

Solution: define a  $\kappa \times \ell$  transformation matrix  $P^{(\kappa)}$  corresponding to the  $\kappa$  largest EOFs (or principal components) of  $Y_N$ , so that  $P^{(\kappa)} \widehat{C}_N P^{(\kappa)^T} = I_{\kappa}$ , and then define the OLS regression equation

$$P^{(\kappa)}y = P^{(\kappa)}X\beta + P^{(\kappa)}u,$$

which leads to an estimator

$$\tilde{\beta} = (X^T P^{(\kappa)} P^{(\kappa)} X)^{-1} X^T P^{(\kappa)} P^{(\kappa)} y$$
(4)

and estimated covariance

$$(X^T P^{(\kappa)T} P^{(\kappa)} X)^{-1}.$$
(5)

### **Refinements (Allen and Tett 1999)**

1. Estimate of covariance likely underestimate, alternative

 $\hat{\text{Var}}\{\tilde{\beta}\} = (X^T \hat{C}_{N_1}^{-1} X)^{-1} X^T \hat{C}_{N_1}^{-1} C_{N_2} C_{N_1}^{-1} X (X^T \hat{C}_{N_1}^{-1} X)^{-1}$  $\hat{C}_{N_1} \text{ estimate of } C_N \text{ from EOF construction, } \hat{C}_{N_2} \text{ independent estimate from separate run of the climate model.}$ 

- 2. Correct for serial correlation
- 3. Effect of random variation in X?
- 4. How to choose  $\kappa$ ? Proposed residual test

$$r^{2} = (y - X\tilde{\beta})^{T} \hat{C}_{N}^{-1} (y - X\tilde{\beta}) \sim \chi^{2}_{\kappa - m},$$

Rejection of  $r^2$  taken as indication that  $\kappa$  is too large.

## Detection of human influence on twentieth-century precipitation trends

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Human influence on climate has been detected in surface air temperature1-5, sea level pressure6, free atmospheric temperature7, tropopause height8 and ocean heat content9. Human-induced changes have not, however, previously been detected in precipitation at the global scale10-12, partly because changes in precipitation in different regions cancel each other out and thereby reduce the strength of the global average signal<sup>13-19</sup>. Models suggest that anthropogenic forcing should have caused a small increase in global mean precipitation and a latitudinal redistribution of precipitation, increasing precipitation at high latitudes, decreasing precipitation at sub-tropical latitudes15,18,19, and possibly changing the distribution of precipitation within the tropics by shifting the position of the Intertropical Convergence Zone<sup>20</sup>. Here we compare observed changes in land precipitation during the twentieth century averaged over latitudinal bands with changes simulated by fourteen climate models. We show that anthropogenic forcing has had a detectable influence on observed changes in average precipitation within latitudinal bands, and that these changes cannot be explained by internal climate variability or natural forcing. We estimate that anthropogenic forcing contributed significantly to observed increases in precipitation in the Northern Hemisphere mid-latitudes, drying in the Northern Hemisphere subtropics and tropics, and moistening in the Southern Hemisphere subtropics and deep tropics. The observed changes, which are larger than estimated from model simulations, may have already had significant effects on ecosystems, agriculture and human health in regions that are sensitive to changes in precipitation, such as the Sahel.

centres directly. We considered three groups of twentieth-century simulations. One group (ANT) includes 27 simulations conducted with 8 models forced with estimates of historical anthropogenic forcing only, including greenhouse gases and sulphate aerosols. A second group (ALL) includes 50 simulations conducted with 10 models forced with estimates of both historical anthropogenic and natural external forcing, including volcanic aerosols and solar irradiance change. A third group (NAT4) includes 15 simulations conducted with 4 models forced with natural external forcing only. Slightly different configurations of historical forcing were used by different modelling centres12. The make-up of each group and the number of simulations used from each model is summarized in Supplementary Table 1. Four models (ECHO-G, HadCM3, MIROC, PCM) contributed simulations to all three groups; the subsets of ANT and ALL simulations from these models are referred to as ANT4 and ALL4 respectively.

We analysed trends in annual zonal mean precipitation anomalies expressed relative to 1961–90. Trends in observed and simulated precipitation were computed and compared quantitatively using the 'optimal fingerprint' method<sup>23,24</sup>, a regression procedure that has been used in many previous detection studies<sup>1</sup>.

Linear precipitation trends from observations and the average of multiple model simulations for 1925–1999 (Figs 1 and 2) exhibit important areas of consistency in the spatial distribution of precipitation change. Both observations and models show that precipitation increased in the Southern Hemisphere deep tropics and subtropics, decreased in the Northern Hemisphere tropics and subtropics, and increased in the Northern Hemisphere poleward of

## Human influence on climate has been detected in surface air temperature, sea level pressure, tropopause height, ocean heat content, etc.....but not, so far, precipitation.

- Models predict...
  - Anthropogenic forcing implies increase in global mean precipitation
  - Increasing at high latitudes
  - Decreasing at sub-tropical latitudes
- We compare increases in land precipitation with changes in 14 climate models.

fourteen climate models. We show that anthropogenic forcing has had a detectable influence on observed changes in average precipitation within latitudinal bands, and that these changes cannot be explained by internal climate variability or natural forcing. We Trends over two periods

- 1925–1999
- 1950-1999

Focus on 40°S–70°N

Climate models:

- ANT (27 simulations from 8 models), anthropogenic signals only (GHGs+aerosols)
- ALL (50 simulations from 10 models), includes volcanic, solar
- NAT4 (15 simulations from 4 models), natural external forcing only

Also look at ANT4, ALL4 (same as ANT, ALL but restricted to same 4 models as are in NAT4)

### Methods

Analyzed trends in annual zonal mean precipitation anomalies relative to 1961–1990. Trends in observed and simulated precipitation compared using "optimal fingerprint" methodology.

Linear trends from observations and averages of multi-model simulations exhibit "important areas of consistency" in spatial distribution of precipitation change. But

- Inconsistency on 20–40°N
- High level of uncertainty



Figure 1 | Comparison between observed (solid black) and simulated zonal mean land precipitation trends for 1925–1999 (left) and 1950–1999 (right). Black dotted lines indicate the multi-model means from all available models (ALL in a and d, ANT in b and e, and NAT as represented by ALL–ANT in c and f), and black dashed-dotted lines those from the subset of

four models that simulated the response to each of the forcing scenarios (ALL4, ANT4 and NAT4). The model-simulated range of trends is shaded. Black dashed lines indicate ensemble means of ALL and ANT simulations that have been scaled (SALL and SANT) to best fit the observations based on a one-signal analysis. Coloured lines indicate individual model mean trends.





**Figure 2** | **1925–1999 changes in observed and simulated precipitation anomalies.** Time series (left panel) of observed annual zonal mean precipitation anomalies in 10° latitude bands (thin black trace) together with ensemble mean annual zonal mean precipitation anomalies in the 50 available ALL simulations (thin blue trace). Straight dashed black and red lines indicate the trends. Green (or yellow) shading identifies latitude bands

with increasing (or decreasing) trends in both observations and models; grey shading indicates disagreement between observed and simulated trends. The map (right panel) indicates the different 10° latitude bands and whether trends agree in sign. Areas with insufficient data are shown in white. Only land precipitation data are used.

# • Estimated combined effects of anthropogenic and natural forcings by regressing observed trends on ahtnropogenic trends

- Response to ALL or ANT was detected in observed trends, but not response to ANT alone.
- To separate contribution from ANT and NAT, need "twosignal attribution analysis"
  - Response to anthropogenic forcing separable from that due to natural forcings and internal variability
  - The two responses "can be reliably separated", though the Allen-Tett for internal residual consistency failed (underestimate of internal variability?)



**Figure 3** | **Results from detection and attribution analysis of zonal precipitation anomalies.** Scaling factors and their 5–95% uncertainty ranges are given from one-signal fingerprint detection analyses for ALL, ANT and NAT4 forced signals as well as subsets of four models, ALL4 and ANT4 (left panel) and from two-signal fingerprint detection analyses (right panel) for ANT and NAT forced signals based on the ALL and ANT ensembles (see

Fig. 1) The residual consistency test<sup>25</sup> passes except where indicated by open circles (test passes after doubling the estimate of internal variability) or closed circles (indicating that the test does not pass even after doubling). Dashed error bars correspond to 5–95% uncertainty ranges when the model simulated variance is doubled.



**Supplementary Figure 7**. Combined uncertainty ranges from 2-signal attribution analysis. 90% uncertainty regions for NAT and ANT scaling factors for the 1950-1999 trends are estimated jointly in a 2-way regression with signals estimated as specified below. The error bars, which cross at the best estimates of the scaling factors, indicate 5%-95% one-dimensional confidence limits. a) 2-way regression with ALL and ANT signals estimated from all available simulations, b) 2-way regression with ALL and ANT, and NAT estimated from the four GCMs that conducted separate ALL, ANT, and NAT simulations.

Detection of anthropogenic influence is robust...

- Little uncertainty about sign of trend
- Robust to different constructions of dataset (e.g. gridding, treatment of missing observations)
- Robust to use of signal patterns from subsets of models
- Consistent with "our understanding of mechanisms"

but...

- Internal variability estimate is smaller than variability estimated from observations
- Discrepancy in magnitude of changes (generally, models underestimate observed trends)

Overall, we find that anthropogenic forcing has had a detectable and attributable influence on the latitudinal pattern of large-scale precipitation change over the part of the twentieth century that we were able to analyse. Our best estimate of the response to anthropogenic forcing suggests (Fig. 1b) that anthropogenic forcing has contributed approximately 50–85% (5–95% uncertainty) of the observed 1925–1999 trend in annual total land precipitation between 40° N and 70° N (62 mm per century), 20–40% of the observed drying trend in the northern subtropics and tropics (0° to 30°N; a decrease of 98 mm per century) and most (75–120%) of the moistening trend in the southern tropics and subtropics (0° to 30°S; 82 mm per century).

### Methods (Allen and Stott 2003)

$$y = \sum_{i=1}^{m} x_i \beta_i + u = X\beta + u,$$
  
$$C_N = E\{uu^T\}.$$

Prewhitening matrix P ( $\kappa \times n$ ),  $E\{Puu^T P^T\} = I_{\kappa}$ .

Estimate  $\tilde{\beta}$  minimizes

$$r^{2}(\tilde{\beta}) = (PX\tilde{\beta} - Py)^{T}(PX\tilde{\beta} - Py)$$
  
=  $\tilde{u}^{T}P^{T}P\tilde{u},$   
 $\tilde{\beta} = (X^{T}P^{T}PX)^{-1}X^{T}P^{T}Py = Fy$ 

Rows of F are "distinguishing fingerprints". Also

$$r_{\min}^2 \sim \chi^2_{\kappa-m}$$

If we ignore uncertainty in estimate of noise variance,

$$\tilde{\beta} \sim N[\beta, V(\tilde{\beta})],$$
  
 $V(\tilde{\beta}) = (X^T P^T P X)^{-1}.$ 

Equivalently,

$$(\tilde{\beta} - \beta)^T (X^T P P^T X) (\tilde{\beta} - \beta) \sim \chi_m^2.$$

Use to construct joint confidence intervals for  $\beta$ .

Also compute (one-dimensional) confidence interval for any linear function  $c^T\beta$ .

### Accounting for uncertainty in estimated noise patterns

Estimation of P based on a set of noise realizations  $\hat{\mathcal{Y}}_1$ .

Take a second independent sample  $\hat{\mathcal{Y}}_2$ .

- v columns of  $\hat{\mathcal{Y}}_2$  not independent
- DF based on  $\widehat{\mathcal{Y}}_2$  is  $v_2 < v$ .
- Typically use overlapping segments, but set  $v_2$  to be 1.5 times the number of nonoverlapping segments

$$\widehat{V}(\widetilde{\beta}) = \frac{F^T \widehat{\mathcal{Y}}_2 \widehat{\mathcal{Y}}_2^T F}{v_2},$$
  
$$(\widetilde{\beta} - \beta)^T \widehat{V}(\widetilde{\beta})^{-1} (\widetilde{\beta} - \beta) \sim mF_{m,v_2},$$
  
$$r_{\min}^2 = \sum_{i=1}^{\kappa} \frac{(P \widetilde{u} \widetilde{u}^T P^T)_{i,i}}{(P \widehat{\mathcal{Y}}_2 \widehat{\mathcal{Y}}_2^T P^T)_{i,i}/v_2} \sim (\kappa - m)F_{\kappa - m,v_2}$$

Final distributional result approximate.

## **Total Least Squares**

Early regression-based approaches to detection and attribution (e.g. Hegerl *et al.* (1996), Allen and Tett (1999)) relied on standard regression equations of the form

$$Y = \sum_{j=1}^{m} \beta_j X_j + \eta \tag{6}$$

where Y is the observational record (e.g. a vector of trend in temperature means),  $X_1, ..., X_m$  are the signals from m climate models, and  $\eta$  is an error term.

Instead of ordinary least squares, Allen and Stott (2003) proposed to fit (6) by *total least squares*, which allows for errors in the  $X_j$ 's as well as Y. Technique extended by Huntingford, Stott, Allen and Lambert (2006).

We reformulate this based on a classical (non-Bayesian) treatment of the errors in variables problem (Gleser 1981).

### Gleser's formulation of errors in variables (EIV)

$$x_i = \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} = \begin{pmatrix} u_{i1} \\ u_{i2} \end{pmatrix} + \begin{pmatrix} e_{i1} \\ e_{i2} \end{pmatrix},$$
(7)

 $x_{i1}$  and  $x_{i2}$  observations of dimensions p and r respectively,  $u_{i1}$ and  $u_{i2}$  are true unobserved signals,  $e_{i1}$  and  $e_{i2}$  noise with covariances  $\sigma^2 I_p$  and  $\sigma^2 I_r$ . Also

$$u_{i2} = B u_{i1}. \tag{8}$$

MLE: choose B and  $u_{i1}, ..., u_{in}$  to minimize

$$Q = \frac{1}{2\sigma^2} \sum_{i} (x_{i1} - u_{i1})^T (x_{i1} - u_{i1}) + \frac{1}{2} \sum_{i} (x_{i2} - Bu_{i1})^T (x_{i2} - Bu_{i1}).$$
(9)

### Computing the MLE

Choose *B* to minimize

$$\tilde{Q} = \frac{1}{2\sigma^2} \sum_{i} (x_{i2} - Bx_{i1})^T (I_r + BB^T)^{-1} (x_{i2} - Bx_{i1}). \quad (10)$$

(also called *generalized least squares* by Gleser)

### **Distribution Theory**

Assume estimator  $\hat{B}_n$  based on n observations,  $U_1$  the  $p \times n$  matrix whose columns are  $u_{i1}, ..., u_{in}$ .

[Assumption A:]  $e_i$  are i.i.d. random vectors with mean 0 and common covariance matrix  $\sigma^2 I_{p+r}$ 

[Assumption C:]  $\Delta = \lim_{n \to \infty} n^{-1} U_1 U_1^T$  exists, positive definite.

[Assumption E:] The cross-moments of the common distribution of the  $e_i$  are identical, up to and including moments of order 4, to the corresponding moments of the multivariate normal distribution with the same mean and covariance matrix.

Then the elements of the  $n^{1/2}(\hat{B} - B)$  have an asymptotic rp-dimensional normal distribution with mean 0 and the covariance between the (i, j) and (i', j') elements is given by  $\sigma^2[\sigma^2\Delta^{-1}(I_p + B^T B)^{-1}\Delta^{-1} + \Delta^{-1}]_{jj'} \cdot [I_r + BB^T]_{ii'}$ .

### **Application to Climate I**

Identify  $x_{i2}$  with Y (single observation, dimension r)

Identify  $x_{i1}$  with  $(X_1, ..., X_m)^T$  (dimension rm). Also

$$B = \left( \beta_1 I_r \quad \beta_2 I_r \quad \dots \quad \beta_m I_r \right). \tag{11}$$

GLSE chooses  $\beta_1, ..., \beta_m$  to minimize

$$S = \frac{(Y - \sum_{j} \beta_{j} X_{j})^{T} (Y - \sum_{j} \beta_{j} X_{j})}{1 + \sum_{j} \beta_{j}^{2}}.$$
 (12)

Equivalent to Allen and Stott (2003). In principle, we could use Gleser's theory to approximate the asymptotic (co-)variances of the estimators. However, here n = 1...

### **Application to Climate II**

Apply Gleser's formulation to individual years. Assume an EOF rotation applied so that  $x_{i2}$  is *r*-dimensional observation for year *i*,  $x_{i1}$  is *rm*-dimensional vector of model runs for year *i*. Assume:

$$x_{i2} = u_{i2} + e_{i2} \quad (r \times 1),$$

$$x_{i1} = u_{i1} + e_{i1} \quad (p \times 1)$$

 $e_{i2} \sim \text{independent with mean 0 and covariance matrix } \sigma^2 I_r,$   $e_{i1} \sim \text{independent with mean 0 and covariance matrix } \sigma^2 I_p,$  $u_{i2} = Bu_{i1}$  where  $B = \left( \beta_1 I_r \ \beta_2 I_r \ \dots \ \beta_m I_r \right).$ 

GLSE chooses  $\beta_1, ..., \beta_p$  to minimize

$$\tilde{Q} = \frac{1}{2\sigma^2} \sum_{i} \frac{(Y_i - \sum_j \beta_j X_{ij})^T (Y_i - \sum_j \beta_j X_{ij})}{1 + \sum_j \beta_j^2}.$$
 (13)

or let B be completely arbitrary??? (Needs rm < n, so we really would have to use a low-order EOF decomposition for this.)

### **Comments / Caveats**

- Gleser's theorem not directly applicable for various reasons, but could adapt.
- Covariance matrix C now needed for individual years, not linear trends.
- Could also examine whether forced model runs are consistent with estimated covariances.
- As presently formulated, doesn't allow for (temporal) autocorrelation.