Long memory or shifting trends in climatological time series*

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1. **Introduction**
   Moberg et al. (2005) used wavelet method to reconstruct NH temperatures by incorporating low resolution proxies, such as lake and ocean sediments together with high resolution data from tree rings. Reconstructed NH temperatures range from 1 AD to 1979, and the main purpose of data analysis is to assess whether it implies *global warming*. Figure 1 shows the plot of the reconstructed annual temperature and sample autocorrelation function (ACF) up to 500 lags. Long cycles and persistent and slow decay of ACF is observed, which naturally leads to long memory model.

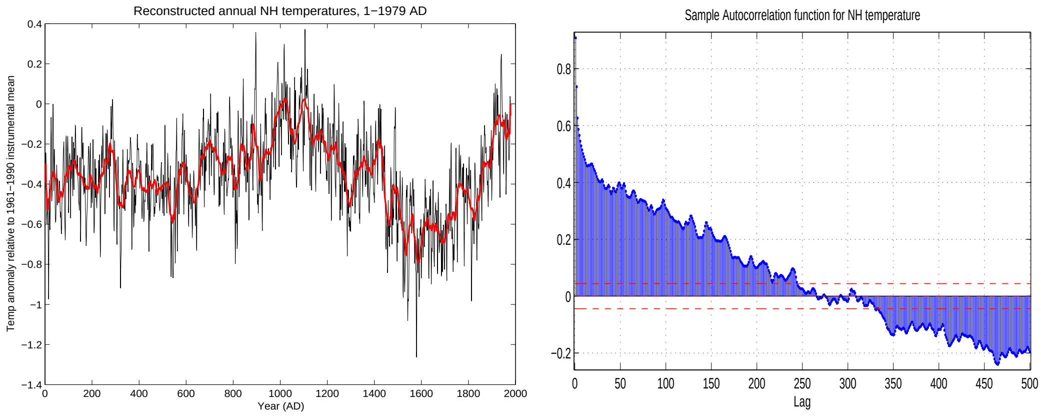

Figure 1: NH temperature data with 30 years moving average (left) and corresponding autocorrelation function (right).

2. **Long memory model - FARIMA**
   Fractionally integrated autoregressive moving average model is fitted to capture power-law decay of ACF function. Parameters are estimated by maximum likelihood method and the order of ARMA parameters are selected from Bayesian information criteria. Best-fitting model is FARIMA(2, .45, 2) with estimates

$$\widehat{d} = .4515(.0354), \quad \widehat{\phi}_1 = .3128(.0510), \quad \widehat{\phi}_2 = -.2577(.0402),$$

$$\widehat{\mu} = -.3295, \quad \widehat{\theta}_1 = -1.5008(.0675), \quad \widehat{\theta}_2 = -.5991(.1085).$$

   Standard residual analysis indicates the goodness of fit of this model.

*Based on "Time series modelling of two millennia of nothern hemisphere temperatures: long memory or shifting trends?" by Terence C. Mills, JRSSA (2007), **170** pp.83-94.

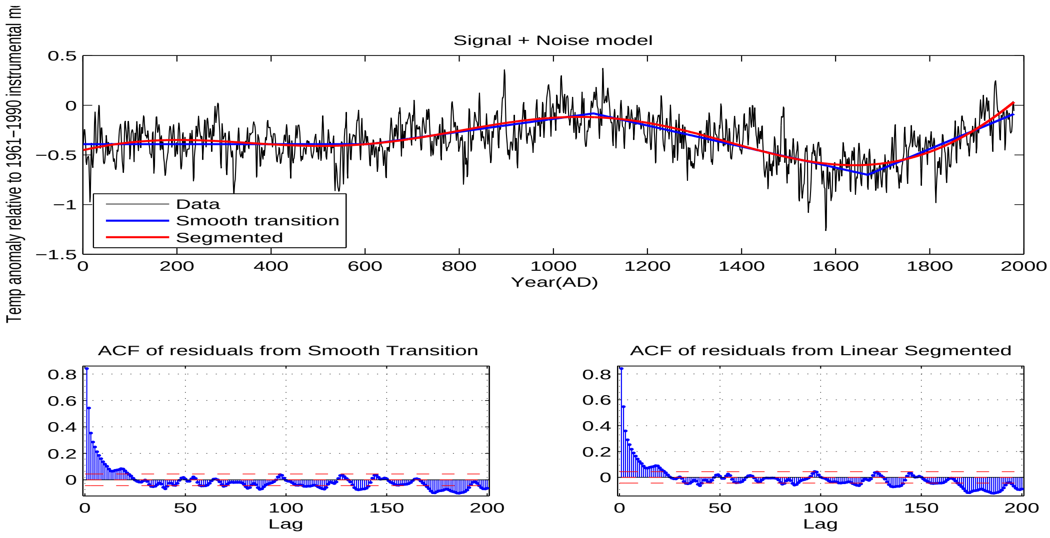

Figure 2: Deterministic trend and noise model. It can be seen from the ACF of residuals that persistent ACF disappeared by considering shifting trends.

3. **Alternative modeling - "trend + noise"**
   Long memory model often confused with structural change, or shifting levels and trends. In this perspective, continuous piecewise linear and the double logistic smooth transition trend is considered in Figure 2. The plot of ACF from residuals indicate that persistent memory successfully diminished.

4. **Tests to discriminate two approaches**
   Both approaches give reasonable modelling of NH temperature data. However, their implications are totally different. Long memory model suggests constant mean that there is no global warming, while structural breaks model implies that there is a significant increasing trend over last 100 years. In order to distinguish stationary long memory model and non-stationary regime switching model, consider the following tests.

   - Ohanissian, Russel and Tsay (2008): This test is based on the invariance property of long memory parameter with respect to temporal aggregation. By using asymptotic normality of GPH estimator, we can perform the χ^2-test for the hypotheses

     $$H_0 : d^{(m_1)} = d^{(m_2)} = \ldots = d^{(m_M)} \text{ vs } H_1 : \text{ not } H_0,$$

     where $d^{(m_i)}$ is the GPH estimator of long memory parameter d with m_i- period nonoverlapping aggregation of data. The test statistic $W = 19.27 \sim \chi^2(7)$, which is very significant. This test concludes that structural break model is plausible.
   - Qu (2008): Test of Qu (2008) is based on the functional central limit theorem of local whittle estimator. Motivation of this test is that trend component only affect estimation up to fairly small number of frequencies used, while true long memory model shows slower and stable estimator over (relatively) larger number of frequencies used. Applying Qu's test to northern hemisphere temperature data yields p-value .1, which implies weaker evidence against long memory model.
   - Müller and Watson (2008): Low-frequency information is the key to explain long memory. Here, authors consider low-frequency information extracted by trigonometric series and perform goodness-of-fit test. The test result for northern hemisphere temperature is mixed. For the hypotheses whether it is unit root or not, test shows weaker evidence

against unit root model (p-value is .1). While, test for the null hypotheses $d = .45$, there is no strong evidence against null hypotheses (p-value is .5).

Considering three above tests, it is not clear whether northern hemisphere temperature can be modeled by either long memory or shifting trend model. It still remains open research question.

5. Conclusion

In this presentation, we have considered time series modelling of two millennia of northern hemisphere temperature reconstructed by Moberg et al. (2005). We examined two important modelling approaches: *i*) (stationary) long memory model and *ii*) (non-stationary) structural breaks plus noise model, which often confused with each other. Both approaches yield reasonable fit to the data, but in the context of global warming, their implications are totally different. Long memory model is essentially weakly stationary model that it implies that there is no global warming. While, structural break model indicates that there is a significant increasing trend over last 100 years. To be more conclusive, we have also performed tests discriminating two approaches. However, the results are differ from test to test that it was impossible to draw solid conclusion. Hence, it still remains open research question.

6. References

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