Proposition 5.15. *As in Proposition 5.10, for a multivariate extreme value distribution G, define*

$$G_*(\mathbf{x}) = G(\psi_1(x^{(1)}), \ldots, \psi_d(x^{(d)}))$$

where

$$\psi_i(x) = (1/(-\log G_i))^{\leftarrow}(x), \qquad x \geq 0, \qquad 1 \leq i \leq d.$$

For a distribution F, define $U_i = 1/(1 - F_i)$, $1 \leq i \leq d$ so U_i has range $[1, \infty]$ and $U_i^{\leftarrow}$ has domain $[1, \infty]$. Set

$$F_*(\mathbf{x}) = F(U_1^{\leftarrow}(x^{(1)}), \ldots, U_d^{\leftarrow}(x^{(d)})), \qquad \mathbf{x} \geq \mathbf{1}.$$

(a) $F_* \in D(G_*)$ iff $1 - F_*$ *is regularly varying on the cone* $(0, \infty)^d$ *with limit function* $-\log G_*(\mathbf{x})/(-\log G_*(\mathbf{1}))$; *i.e., for* $\mathbf{x} > \mathbf{0}$

$$\lim_{t \to \infty} (1 - F_*(t\mathbf{x}))/(1 - F_*(t\mathbf{1})) = (-\log G_*(\mathbf{x}))/(-\log G_*(\mathbf{1})). \qquad (5.35)$$

(b) $F \in D(G)$ iff *marginal convergences* (5.16) *hold and* $F_* \in D(G_*)$.