Define $\psi(t)$ for $t<0$ so that $\frac{\psi(t)}{t} \to 0$ as $t \uparrow 0$.

$$\frac{1-F(t+x\psi(t))}{1-F(t)} = \exp\left\{\frac{1}{t+x\psi(t)} - \frac{1}{t}\right\}$$

$$= \exp\left\{\frac{-x\psi(t)}{t(t+x\psi(t))}\right\} \to e^{-x}$$

if we define $\psi(t) = t^2$.

So limit is Gumbel dist^n.

[Argument over "probable max precipitation"]

An example of a continuous distribution not in any d.o.a.:

$$1-F(x) = \frac{1}{\log x} \quad \text{as} \quad x \uparrow \infty.$$

$$\frac{1-F(tx)}{1-F(t)} = \frac{\log t}{\log t + \log x} \to 1 \quad \text{not of form } x^{-\alpha}$$

and de Haan's condition would require $\int_t^\infty (1-F(s))\,ds$

$$= \int_t^\infty \frac{1}{\log s}\,ds < \infty \quad \text{but this is not true.}$$

2nd 2/11/25 Discrete Distributions Need a separate theory.