STOR 834: EXTREME VALUE THEORY

Richard L. Smith
Advanced Special Topics Course

Tuesdays and Thursdays 12:30-1:45, Hanes 125

Background: Extreme Value Theory is the branch of probability theory and statistics that is concerned with extremes of sequences of random variables and stochastic processes. The subject started in the 1920s with a seminal paper by Fisher and Tippett and an almost-parallel contribution by Fréchet, but was first put on a firm mathematical ground by Gnedenko (1943). The original theory was for independent and identically distributed (IID) random variables but it was quickly extended to other cases such as independent non-identically distributed random variables, stationary (dependent) sequences and continuous-parameter stationary processes. Beginning in the 1950s but more prominently in the 1970s, the theory started to be extended to multivariate random variables and more recently to general stochastic processes including spatial and spatio-temporal processes. Statistical methods for extremes were originally developed in a classic but now outdated book by Gumbel (1958), but were developed much more extensively in the 1980s with the increased availability of computers and automated computational algorithms (e.g. fitting the Generalized Extreme Value distribution by maximum likelihood). These days, the theory is widely applied in many fields but three of the main ones are reliability and strength of materials, mathematical finance and insurance, and environmental (especially, climate) extremes. A side interest of mine is applications to records in athletic events.

The aim of the course is twofold – on the one hand, to cover the mathematical theory (probabilistic and statistical) with some degree of rigor, and on the other, some of the major areas of application. The mathematical theory will cover broadly four categories – IID sequences, stationary sequences, multivariate and spatial – and the applied fields will include each of the reliability, insurance/finance and environmental/climate fields. The last of these has been my own main applied statistics interest in recent years so we will certainly be covering that topic in some detail.

Instructor. Richard L. Smith, Hanes 303, rls@email.unc.edu (email is the preferred method of communication). Tentatively, I plan to hold office hours each week on Mondays from 1-3 pm, but will discuss this with the class before finalizing that. Any changes will be announced. There will not be an IA or grader for this course.

Class Materials. Materials for the course will be in two places: the course website at

https://rls.sites.oasis.unc.edu/s834-2023/s834.html

and the Canvas page that you should all have access to (https://edtech.unc.edu/service/canvas/ and login using your onyen). I have decided to use Canvas rather than Sakai as it will be a good way for me to learn about the features of Canvas before I have to do so in the more high-stress context of an undergraduate class. Mainly, I will put new materials on the publicly accessible website, but Canvas does have the very useful feature of a "Course Reserves" tab. I have requested some books on extreme value theory to be placed there as reserve material, and as I refer to research papers during the course, I will most likely place the papers in this section for you to refer to.

Schedule. The first class is on Tuesday, January 10, and the last class on Thursday, April 27. There will be no class on Tuesday, February 14 or Thursday, April 6 (university-designated "wellness days"). In addition, the week of March 13-17 is Spring Break so we shall not have class on March 14 or 16. With those exceptions, there will be a class every Tuesday and Thursday unless cancelled by me.

Attendance is expected at all classes but this is largely on an honor system. There is no need to ask my permission if you happen to miss occasional classes for either personal or professional reasons. If you are likely to be away for an extended period (for instance, three or more consecutive classes), please get in touch and we will discuss alternative arrangements if needed. If I notice you are frequently absent, I may follow up with you about the reasons for that.

I have posted a tentative schedule of classes (i.e. what material we will be covering on what date) at https://rls.sites.oasis.unc.edu/s834-2023/ScheduleOfClass.pdf, but as of now, this is very tentative – expect frequent updates!

Assessment. There will be no written exams in this course. The main assignment will be a short project due at the end of semester. I am flexible about the format of this project but my main expectation is that it will be either (a) a review of literature or some other topic outside what I present in class, or (b) some independent research (which could be data analysis) of your own. I will expect you to present a report on your project in class and to hand in a short (4-6 pages) report about it. If you attend the class with reasonable regularity and complete the project you will get a P grade in the class. H grades will be awarded at my discretion to students who do a superior job on the project or in other ways make an exceptional contribution to the course.

Initially I announced there would not be any homeworks either but I am now having a rethink about that and will discuss it with the class. I feel it would be useful to have a few low-key theoretical exercises and data analyses woven into the course material, but I still plan to be flexible about how I do this: you will earn extra credit for completing the homeworks, but I still plan to make the end of semester presentation the main requirement based on which I will assign grades.

Outline of topics:

1. The Extreme Value Limit Distributions. The basic theory of extremes of univariate IID random variables as formulated by Fisher and Tippett, Gnedenko, and de Haan. The extreme value distributions and their domains of attraction; rates of convergence; statistical theory of the Generalized Extreme Value and Generalized Pareto distributions.
2. Extremes in Stochastic Sequences. Leadbetter's D and D' conditions; the extremal index; computations of extremal index in non-trivial situations, including Markov chains. Estimation of extremes under dependent conditions.
3. Multivariate Extremes. Multivariate extreme value distributions and their domains of attraction. Asymptotic dependence and asymptotic independence. Estimation under both parametric and semiparametric families.
4. Spatial Extremes. Max-stable and max-infinitely divisible processes; constructions and characterizations. Special classes such as Brown-Resnick processes. Statistical theory: method of composite likelihood; progress towards exact maximum likelihood; Bayesian approaches.
5. Applications to Reliability and Strength of Materials.
6. Applications to Finance and Insurance.

7. Applications to Climate and Environment.

References:

The following are well-established references be placed in the Course Reserves section in Canvas (as of 1/9/23, de Haan and Ferreira is available in electronic format and you can access it immediately; the other three are on order as E-books):

Coles, S. (2001), *An Introduction to Statistical Modeling of Extreme Values*. Springer, New York.

Haan, L. de and Ferreira, A. (2006), *Extreme Value Theory: An Introduction*. Springer, New York.

Leadbetter, M.R., Lindgren, G. and Rootzén, H. (1983), *Extremes and Related Properties of Random Sequences and Series.* Springer-Verlag, New York.

Resnick, S. (1987), *Extreme Values, Point Processes and Regular Variation*. Springer-Verlag, New York.

Other Books on Extreme Value Theory:

Beirlant, J., Goegebeur, Y., Segers, J. and Teugels, J. (2004), *Statistics of Extremes: Theory and Applications*. Wiley, Chichester, England.

Cramér, H. and Leadbetter, M.R. (1967), *Stationary and Related Stochastic Processes*: *Sample Function Properties and Their Applications.* Wiley, New York.

Embrechts, P., Klüppelberg, C. and Mikosch, T. (1997), *Modelling Extremal Events for Insurance and Finance*. Springer-Verlag, New York.

Gumbel, E.J. (1958), *Statistics of Extremes*. Columbia University Press, New York.

Resnick, S. (2007), *Heavy-Tail Phenomena: Probabilistic and Statistical Modeling*. Springer, New York.

R.L. Smith (2003), Statistics of extremes, with applications in environment, insurance and finance. Chapter 1 of *Extreme Values in Finance, Telecommunications and the Environment*, edited by B. Finkenstadt and H. Rootzén, Chapman and Hall/CRC Press, London, pp. 1-78 (the pre-publication version of this chapter is on my website, https://rls.sites.oasis.unc.edu/postscript/rs/semstatrls.pdf).

Related Activities at UNC:

Sid Resnick was one of the pioneers of multivariate extreme value theory (and much else…), and his 1987 book is one of the classic references in this field. Sid is giving a series of lectures at UNC this spring as part of the department's RTG activities, starting on the first day of our class, January 10 2023. The announcement is on the department webpage, https://stor.unc.edu/news-item/rtg-lectures-by-resnick/.

Also, the department is hosting a memorial conference for our recently deceased faculty member Professor Ross Leadbetter, whose two books (with Lindgren and Rootzén in 1983, and his earlier book with Harald Cramér) are also among the classic references in this field of research. The date of this event is Saturday, February 25, and you can see full details at https://stor.unc.edu/news-item/a-celebration-in-memory-of-ross-leadbetter/.

Although neither of these events is a formal part of the course, I strongly encourage you to attend both of them.

Pro Forma Items:

Accessibility Resources

If you require accommodations for any reason and do not feel comfortable discussing these directly with me, you are encouraged to approach the Office of Accessibility Resources and Service (ARS), https://ars.unc.edu or email ars@unc.edu.

Counseling and Psychological Services (CAPS)

CAPS is strongly committed to addressing the mental health needs of a diverse student body through timely access to consultation and connection to clinically appropriate services, whether for short or long-term needs. Their website is https://caps.unc.edu/.

Title IX Resources

Any student who is impacted by discrimination, harassment, interpersonal (relationship) violence, sexual violence, sexual exploitation, or stalking is encouraged to seek resources on campus or in the community. Relevant offices are the Director of Title IX Compliance (Adrienne.allison@unc.edu), Report and Response Coordinators in the Equal Opportunity and Compliance Office (reportandresponse@unc.edu), Counseling and Psychological Services (confidential), or the Gender Violence Services Coordinators (gvsc@unc.edu; confidential) to discuss your specific needs. Additional resources are available at safe.unc.edu.

Honor Code

For the complete honor code, please visit http://instrument.unc.edu/. The main “honor code” issue with this class is to remind you that work that you present as your own must truly be your own work and any use of outside sources should be cited. I have no objection to students working together on assignments but the final work you hand in must be your own (unless explicitly acknowledged as a collaboration between two or more students).

COVID and Masking

There is no longer any requirement that students wear masks or practice any other form of social distancing. However, there are frequently reports of new variants as well as other infectious diseases (RSV, flu) in our community. I encourage students to continue to wear masks in indoor settings where it is safe and appropriate to do so. For myself, I do not intend to wear a mask while lecturing but I may do so for other in-person meetings.

Administrative details

Any questions regarding course registration should be directed to Ms. Christine Keat, crikeat@email.unc.edu. The instructor reserves the right to make changes to the syllabus.