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Suppose we have a linear model $Y = X\beta + \epsilon$ with $\epsilon = \begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_n \end{pmatrix}$ and $\epsilon_1, ..., \epsilon_n$ uncorrelated with mean 0 and common variance σ^2 (in other words, the usual assumptions). As seen in class, the least squares estimator of β is $\hat{\beta} = (X^T X)^{-1} X^T Y$.

Prove the *Gauss-Markov theorem* in the following form. Suppose we are interested in estimating a scalar parameter $\theta = a^T \beta$ for some given $p \times 1$ vector a. The obvious estimator is $\hat{\theta} = a^T \hat{\beta}$ and it is easily checked that this is unbiased. Suppose $\tilde{\theta} = c^T Y$ (where c is a $n \times 1$ vector) is some other linear estimator that is also unbiased. Prove that $Var(\tilde{\theta}) \geq Var(\hat{\theta})$ with equality only if $\tilde{\theta} = \hat{\theta}$.