STOR 151 SECTION 1 FINAL EXAM
DECEMBER 11 2018

YOUR NAME:

PID:

Honor pledge: On my honor, I have neither given nor received unauthorized aid in this exam.

SIGNATURE:

Please write your answers in a blue book, except for the graph in Question 7 which is at the end of this exam. **Please hand in this question paper as well as your blue book.**

This is an open book exam. Course text, personal notes and calculator are permitted. You have 3 hours to complete the test. Personal computers and cellphones are not allowed. If you have any queries about the meaning of a question, ask the instructor for advice.

SHOW ALL WORKING — even correct answers will not get full credit if it's not clear how they were obtained. Incorrect answers will gain substantial credit if the method of working is substantially correct.

Answer six of the eight questions. If you attempt more than six, all the answers will be graded but only the best six (complete questions) will count. Each question is worth a total of 20 points and the whole exam is worth 120 points (which will be rescaled to a maximum of 40 for grading purposes). Points for each individual part of a question are also given in square brackets.

Questions may be answered in any order.

1. An irregular eight-sided die has the following probabilities:

Number on face of the die	1	2	3	4	5	6	7	8
Probability	0.1	0.1	0.1	0.1	0.1	0.1	x	x

(a) Find x. [**3 points**]

(b) Let A be the event "the die shows an odd number", Let B be the event "the die shows one of 1, 2, 3, 4". Find $P(A)$, $P(B)$, $P(A \text{ and } B)$ and $P(A \text{ or } B)$. Are the events A and B independent? [**7 points**]

(c) Suppose the die is tossed three times. What is the probability that the total is greater than 22? [**5 points**]

(d) If the same die is thrown a large number of times, the mean score is 5.1 and the standard deviation is 2.39 (you can assume these numbers without proof). If the die is thrown 100 times, what is the approximate probability that the *average* score is at least 5.5? [**5 points**]

2. A new test for type-2 diabetes classifies each patient on a 3-point scale, where a score of 1 means the patient is highly likely to have type-2 diabetes, a score of 3 means the patient is unlikely to have type-2 diabetes, and a score of 2 is intermediate. For a patient who has type-2 diabetes, the probabilities of a score of 1, 2 or 3 are respectively 0.6, 0.25, 0.15. For a patient who does not have type-2 diabetes, the probabilities of a score of 1, 2 or 3 are respectively 0.1, 0.1, 0.8. Within the population being tested, 8% of all patients have type 2 diabetes.

   (a) Represent this information in the form of a tree diagram. What is the probability that a randomly chosen patient both has diabetes and gets a score of 1? [**5 points**]

   (b) What is the overall probability that a patient gets a score of 2 or 3? [**5 points**]

   (c) Given that a patient has a score of 1, what is the probability that he/she has type 2 diabetes? [**5 points**]

   (d) A physician decides that it is too complicated for him to explain probabilities to his patients so he simply tells every patient with a score of 1 or 2 that they have type 2 diabetes, and every patient with a score of 3 that they do not. What proportion of all patients are given the wrong diagnosis by this physician? [**5 points**]

3. A large survey of 17-year-old boys in the USA included the question "Do you work out with weights at least twice a week?" 71% of the respondents said yes. Assume that this proportion applies to the full population of 17-year-old boys in the USA.

   (a) Among a sample of 10 boys, what is the probability that at least 8 of them work out with weights at least twice a week? [**5 points**]

   (b) Among a sample of 31 boys, what is the probability that exactly 22 (71% of 31) of them work out with weights at least twice a week? [**5 points**]

   (c) Among a sample of 121 boys, what is the approximate probability that at least 91 of them work out with weights at least twice a week? [**5 points**]

   (d) Now suppose I tell you that the sample in part (c) actually came from Germany, i.e. not part of the original survey, and that the sample reported exactly 91 boys (of the 121) who do work out with weights at least twice a week. Does this sample provide evidence that 17-year-old boys in Germany have different exercise habits from those in the USA? State carefully what assumptions you are making and how you reached your conclusion. [**5 points**]

4. A sample of ten 15-year-old girls from public schools in North Carolina has a mean height of 64.1 inches with a sample standard deviation of 2.7 inches.

   (a) Calculate a 90% confidence interval for the mean height of all 15-year-old girls from public schools in North Carolina. [**7 points**]

   (b) Test the null hypothesis that the mean height of all 15-year-old girls from public schools in North Carolina is 65.8 inches. Use a two-sided test at significance level 0.05 and state any assumptions you make. [**8 points**]

(c) Suppose the true (population) standard deviation, for the mean height of all 15-year-old girls from public schools in North Carolina, is 3.2 inches. How large a sample size would you need for the total width of the confidence interval in (a) to fall below 1 inch? [**5 points**]

5. At the start of a new track season, the coach has every member of his team run a 400 meter time trial. Six weeks later, he repeats the trial. These are the recorded times, to the nearest tenth of a second:

Runner	A	B	C	D	E	F	G	H	I	Mean	SD
Trial 1	53.3	54.3	51.5	51.9	52.6	51.3	50.2	54.9	51.0	52.333	1.571
Trial 2	52.6	53.4	51.3	51.5	52.1	51.1	50.3	53.8	50.9	51.889	1.178
Difference	0.7	0.9	0.2	0.4	0.5	0.2	–0.1	1.1	0.1	0.444	0.394

(a) Analyze the data as a two-sample experiment. Is there a statistically significant improvement between the running times in Trials 1 and 2? Use a one-sided test with significance level 0.05. [**7 points**]

(b) Analyze the data as a paired comparison experiment. Is there a statistically significant improvement between the running times in Trials 1 and 2? Use a one-sided test with significance level 0.05. [**6 points**]

(c) Which of the two tests do you think is most appropriate in this case? Explain why. [**2 points**]

(d) For the paired comparison experiment, compute a 99% confidence interval for the mean improvement in a runner's time between Trial 1 and Trial 2. [**5 points**]

6. A recent study from the Wake Forest School of Law produced the following data on race and jury selection:

Status of Juror	Race			
	White	Black	Other	Unknown
Retained	10,402	2,628	324	3,389
Removed by Judge	1,729	574	133	841
Removed by Prosecutor	1,437	755	94	716
Removed by Defense	2,960	288	63	876
Removed, Source Unknown	1,351	427	36	600
Total	17,879	4,672	650	6,422

(a) The data show that, out of 17,879 white prospective jurors, 10,402 were retained (in other words, actually served on a jury). Out of 4,672 black prospective jurors, 2,628 were retained. Does this indicate a statistically significant difference in the retention rates of whites and blacks? [**8 points**]

(b) If we focus just on jurors who were either white or black (removing the "Other" and "Unknown" categories) and who were removed by one of judge, prosecutor or defense, we get the following table:

	White	Black	Total
Removed by Judge	1,729	574	2,303
Removed by Prosecutor	1,437	755	2,192
Removed by Defense	2,960	288	3,248
Total	6,126	1,617	7,743

Does this table indicate a racial difference in removal rates among judges, prosecutors and defense attorneys? [**8 points**]

(c) Write a short paragraph explaining your conclusions in plain English, noting any differences of interpretation between your answers to (a) and (b). [**4 points**]

7. Have the number of freezing days in December in North Carolina gone down as a result of global warming? The figure on the back page of this exam shows a scatterplot of the number of December days for which the daily average temperature at RDU airport is less than 32 degrees Fahrenheit, against year, for the period 1944–2017. For convenience and numerical stability, the variable "year" is recoded to years since 1943, so year 1 is 1944 and year 74 is 2017. You can assume the following: $R = -0.331,\ \bar{x} = 37.5,\ \bar{y} = 3.703,\ s_x = 21.51,\ s_y = 3.423$.

   (a) Find the regression line in the form $y = b_0 + b_1 x$. What are b_0 and b_1? [**7 points**]

   (b) Draw the regression line on the plot. To be specific: go to the last page of the exam, draw the line, and hand this in with your blue book. Don't forget to write your name on the sheet! (Exact precision is to required: what I am looking for is a visual impression of approximately where the line should go.) [**3 points**]

   (c) Based on the regression line fitted to 1944–2017, what is your prediction for the number of days with daily mean temperature below 32 degrees in 2018 (year 75)? In 2040 (year 97)? [**3 points**]

   (d) In what year will the regression line fall below zero? [**3 points**]

   (e) State your conclusions. Do you think the assumptions of a linear regression analysis are valid in this case? Can you suggest a better method of analyzing the data? [**4 points**]

8. At the time of preparing this exam, the Congressional election in the Ninth District of North Carolina was undecided. Central to the dispute is an allegation concerning absentee ballots in two counties, Bladen and Robeson. Below are the numbers of absentee ballots requested, and numbers of absentee ballots successfully returned, in each of the eight counties that comprise the Ninth District. (Note that most of Mecklenburg County, which includes Charlotte, is in another congressional district, but the statistics from which these numbers are taken are compiled by county not by congressional district.)

County	Number Requested	Number Returned	Percent Not Returned
SCOTLAND	6698	6565	2.0
MECKLENBURG	216175	211856	2.0
UNION	53604	52260	2.5
ANSON	4000	3857	3.6
RICHMOND	7197	6987	2.9
CUMBERLAND	50782	49168	3.2
BLADEN	8110	7193	11.3
ROBESON	16069	14301	11.0

As can be seen, Bladen and Robeson counties showed a substantially higher rate of non-return than the other six. The allegations essentially center on the possibility that these ballots were misappropriated by party workers.

(a) For Bladen and Robeson counties combined, there were 24,179 absentee ballots requested of which 2,685 were not returned. If p_1 represents the proportion of non-returned absentee ballots within these two counties, find a 95% confidence interval for p_1. [**4 points**]

(b) Let us look at two of the counties that are not under suspicion, Anson and Richmond. Is there a statistically significant difference in the proportion of non-returned absentee ballots between those two counties? Explain carefully how you reach your conclusions and any assumptions you make. [**4 points**]

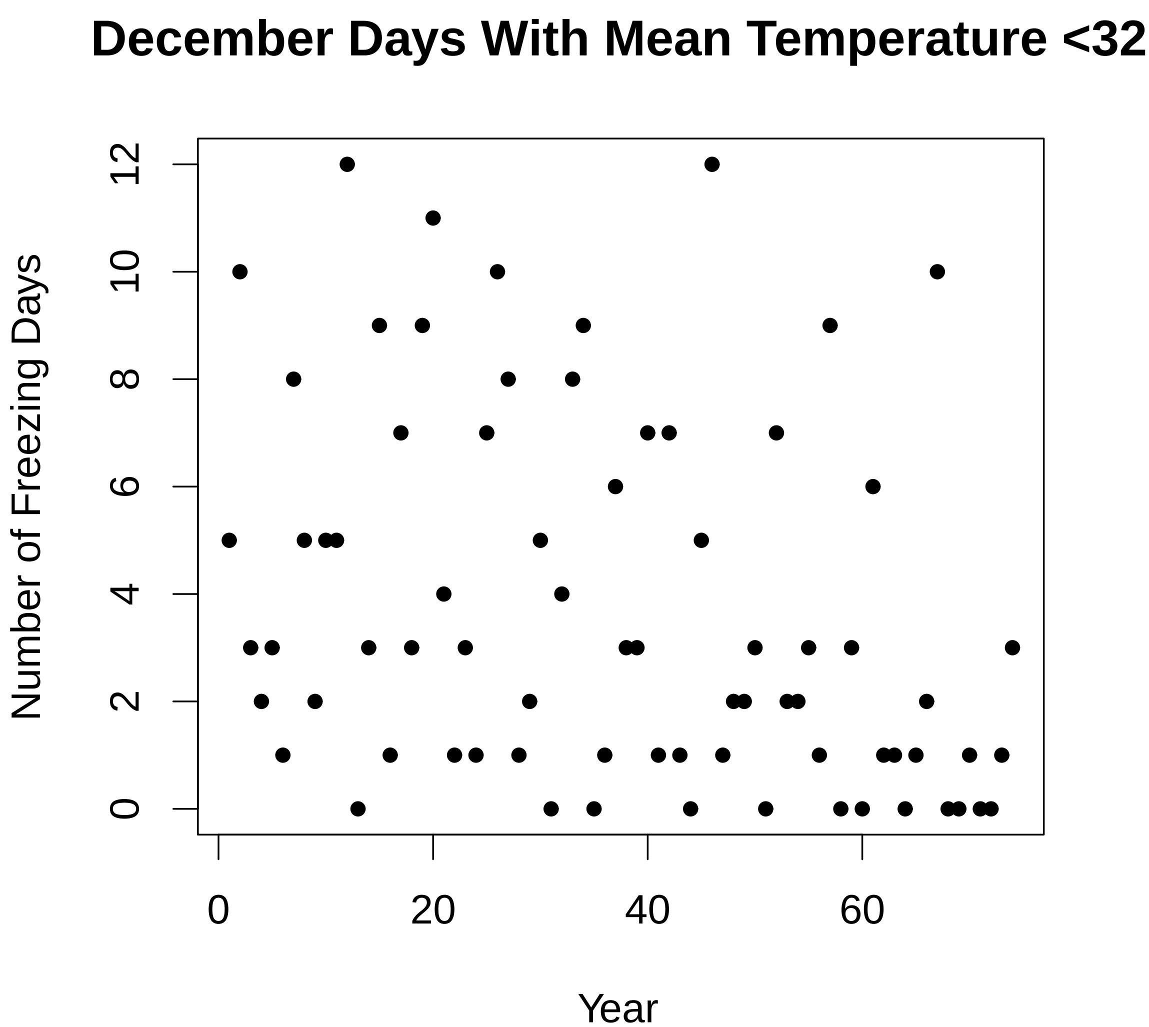
(c) Combining Scotland, Union, Anson, Richmond and Cumberland counties (but omitting Mecklenburg because most of it is in another district), there were 122,281 absentee ballots requested of which 2,994 (2.45%) were not returned. Combined with your answer to (a), does this provide clear evidence that the rate of non-return of absentee ballots in Bladen and Robeson counties was higher than in the other five counties? Explain your reasoning. [**4 points**]

(d) Based on the above numbers, it looks as though about 2,100 ballots went missing in Bladen and Robeson counties. Overall, the Democrat candidate, Dan McCready, got about 60% of all mail-in absentee ballot votes (a substantially higher proportion than the in-person ballot). If that 60% proportion for McCready also applied to the missing ballots, what would be the mean and standard deviation of the number of votes for McCready that went missing? Explain your reasoning and state any assumptions you make. [**5 points**]

(e) At the time of writing, the Republican candidate Mark Harris holds a 905 vote lead in the race, which the Board of Elections has declined to certify because of the alleged irrgularities. Based on the figures given in (d), do you think there is a reasonable chance that McCready would overhaul Harris if all the missing votes were found and counted? (*Note:* There is also a third party candidate in the race but please ignore that for the purpose of this question.) [**3 points**]

HAND IN THIS PAGE WITH YOUR ANSWER

YOUR NAME:



SOLUTIONS AND COMMENTS

1. (a) The probabilities must add to 1, so $x = 0.2$.

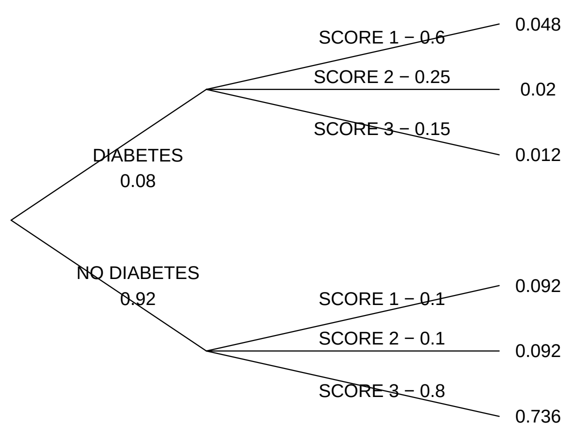
   (b) 0.5, 0.4, 0.2, 0.7. We have $P(A$ and B$) = 0.2 = 0.5 \times 0.4 =$ P(A) $\times$ P(B), so A and B *are* independent.

   Added after grading: Some students just *assumed* A and B were independent, which isn't correct. The event (A and B) occurs when the die shows 1 or 3; the probability of that is $0.1 + 0.1 = 0.2$. Apart from that, the most common error was to assume the eight sides were all equally likely, which leads to a quite different set of answers.

   (c) Possible combinations are three 8s (total 24), or two 8s and one 7 (total 23). The latter could be arranged one of three ways ((8,8,7) or (8.7.8) or (7,8,8)). The corresponding probabilities are 0.2^3 and 3×0.2^3 so the answer is $4 \times 0.2^3 = 4 \times 0.008 = 0.032$.

   (d) For $n = 100$, the standard deviation of $\bar{x}$ is $\frac{2.39}{\sqrt{100}} = \frac{2.39}{10} = 0.0239$, so for a normal probability calculation, $z = \frac{5.5-5.1}{0.239} = 1.67$. The left tail probability is 0.9525 and therefore the right tail probability is $1 - 0.9525 = 0.0475$. This relies on the Central Limit Theorem which is valid in this case because $n > 30$. (Quite a few students forgot to divide by 10 to get the standard deviation of $\bar{x}$, the generic formula for which is $\frac{\sigma}{\sqrt{n}}$.)

2. (a) See the following:



   The probability that a patient both has diabetes and gets a score of 1 is $0.08 \times 0.6 = 0.048$. The other probabilities down the right hand side are calculated similarly.

   (b) $0.02 + 0.012 + 0.092 + 0.736 = 0.86$.

   (c) $\frac{0.048}{0.048+0.092} = 0.343$.

   (d) This corresponds to the event: either the patient has type 2 diabetes, and gets a score of 3, or the patient does not have type 2 diabetes, and gets a score of 1 or 2. The combined probability is $0.012 + 0.092 + 0.092 = 0.196$.

3. (a) $\binom{10}{8}(0.71)^8(0.29)^2 + \binom{10}{9}(0.71)^9(0.29) + (0.71)^{10} = 0.2444 + 0.1330 + 0.0326 = 0.410$. Here, as with all these questions, small deviations from the stated answer due to rounding error will not be penalized. Note that $\binom{10}{8} = \frac{10!}{8!2!} = \frac{10\times9}{2} = 45$, while $\binom{10}{9} = 10$.

(b) $\binom{31}{22}(0.71)^{22}(0.29)^9 = 20,160,075 \times (0.71)^{22} \times (0.29)^9 = 0.1562$. *Side note.* $\binom{31}{22} = \frac{31!}{22!9!}$. You should be able to evaluate the factorials on your calculator but the alternative way to do it is $\frac{31\times30\times29\times28\times27\times26\times25\times24\times23}{9\times8\times7\times6\times5\times4\times3\times2\times1} = \frac{31\times30\times29\times28\times27\times26\times25\times24\times23}{362,880} = 20,160,075$.

(c) Use the normal approximation to the binomial distribution (preferably, with the continuity correction). The mean and standard deviation are $\mu = 121 \times 0.71 = 85.91$ and $\sigma = \sqrt{121 \times 0.71 \times 0.29} = 4.99$ so for a normal probability calculation (for "greater than or equal to 91", which we also interpret as "greater than 90.5") we have $z = \frac{90.5-85.91}{4.99} = 0.92$. The left tail probability is 0.8212 and therefore the right tail probability (which we want) is $1 - 0.8212 = 0.1788$. *Alternative solutions:* Some students misinterpreted the problem as *exactly 91* rather than *at least 91* — if you work that out using the binomial formula, the answer is 0.04897694. I have partial credit for that though it wasn't the answer wanted. At least one student tried to use the binomial formula for each of 91, 92,...,121 and add them all up to get the answer to the question asked, but that's a lot of work to do on a pocket calculator and in fact the student concerned did not get the right answer. However, if anyone wants to know, the exact answer (computed in R — pbinom(90.5,121,.71,lower.tail=F)) is 0.1794476.

(d) This is a hypothesis testing problem. Suppose p is the proportion of all 17-year-old boys in Germany who work out with weights at least twice a week. We want to test $H_0: p = 0.71$ against $H_A: p \neq 0.71$. The null hypothesis corresponds to the proportion in Germany being exactly the same as in the USA. Here, 91 is the number of boys in the sample who say yes to the question asked (whether they work out with weights at least twice a week). Under H_0, the mean of that quantity is 85.91. Since $91 > 85.91$, the one-sided p-value is the probability that the number of positive responses is 91 or greater when the null hypothesis is true. By the answer to (c), that probability is approximately 0.1788. Since it's a two-sided test, the two-sided p-value is twice that, or 0.3576. This is clearly not a statistically significant conclusion so we conclude there is no evidence that 17-year-old boys in Germany have different exercise habits from 17-year-old boys in the USA. (Some students interpreted the question differently, assuming that the American sample was also of size 121 and testing $p_1 = p_2$ using the methods of Chapter 6. This leads to very similar answers and I did give credit for that.)

4. (a) The standard error is $\frac{2.7}{\sqrt{10}} = 0.8538$. Based on a t distribution with 9 degrees of freedom (look up "two tails, 0.100" in the header), $t^* = 1.83$. Therefore, the 90% confidence interval is $64.1 \pm 1.83 \times 0.8538 = (62.5, 65.7)$ rounded to one decimal place.

(b) $\frac{65.8-64.1}{0.8538} = 1.99$. For a two-tailed test with significance level 0.05, the critical value is 2.26. Because $1.99 < 2.26$, we accept the null hypothesis: there is no evidence that the population mean is different from 65.8. The assumptions used here are that the sample is independent and that the overall distribution of students' heights is reasonably close to a normal distribution, which in practice means no extreme skewness or outliers.

(c) If we repeated the confidence interval calculation from (a) with known standard deviation, we would use a normal distribution and therefore replace t^* by 1.645. The total length of the confidence interval in this case would therefore be $2 \times 1.645 \times \frac{3.2}{\sqrt{n}}$ which we require to be less than 1. Therefore, $n > (2 \times 1.645 \times 3.2)^2 = 110.8$. Rounding up, the sample size needs to be 111 or larger. (Some students continued to use $t^* = 1.83$, following part (a), which leads to $n = 138$ — since the question didn't specify exactly what value of t^* you were to use, I also gave full credit for that answer.)

5. (a) For a two-sample test, the unpooled standard error is $\sqrt{\frac{1.571^2}{9} + \frac{1.178^2}{9}} = 0.6545$ so the test statistic is $\frac{0.444}{0.6545} = 0.6784$. Since this is less than 1 there is no need to look up the t tables — it is clearly not a statistically significant result.

   (b) For the paired test we take the differences (0.7, 0.9, etc.) as the raw data (to answer a question one student asked me, we do take account of sign, in particular –0.1 for runner G) and the mean is 0.444, standard deviation is 0.394. The standard error is $\frac{0.394}{\sqrt{9}} = 0.1313$ and the t statistic is $\frac{0.444}{0.1313} = 3.38$. This is a t test with $df = 8$, using a one-tail probability of 0.05. From the table, $t^* = 1.86$. In this case there is a clear difference, $3.38 > 1.86$, so we *reject* the null hypothesis that there has been no improvement (in fact it's very close to a one-tailed p-value of 0.005, for which $t^* = 3.36$).

   (c) The paired t test is appropriate because it's the same group of runners. The two-sample test would be appropriate if the second trial was an independent trial with a different group of runners. The conclusion is that the runners have made a clear improvement over the weeks between the two trials.

   (d) For 99% confidence, we use the column in the t-table headed "two tails, 0.010," for which $t^* = 3.36$ as just noted. The confidence interval is then $0.444 \pm 3.36 \times 0.1313 = (0.003, 0.885)$. Since the times are only recorded to 0.1 second accuracy, the sensible way to report the answer would be that we have 99% confidence that the improvement is between 0 and 0.9 seconds.

6. (a) Let p_1 be the rate of retention for whites and p_2 be the rate of retention for blacks. Then $\hat{p}_1 = 10402/17879 = 0.5818$ with a standard error $\sqrt{0.5818 \times (1 - 0.5818)/17879} = 0.00369$, and $\hat{p}_2 = 2628/4672 = 0.5625$ with a standard error $\sqrt{0.5625 \times (1 - 0.5625)/4672} = 0.00726$. The standard error for the difference is $\sqrt{0.00369^2 + 0.00726^2} = 0.0081$. Alternatively, we could use the pooled $\hat{p} = \frac{10402+2628}{17879+4672} = 0.5778$ which also leads to standard error 0.0081. The z statistic for $H_0 :\ p_1 = p_2$ is $\frac{0.5818-0.5625}{0.0081} = 2.38$, for which the two-sided p-value is $2 \times (1 - 0.9913) = 0.0174$. Thus, the difference is statistically significant at level 0.05 but not at level 0.01; not "overwhelmingly" significant. This distinction matters for the later discussion.

   (b) The table given in the question is the "Observed" table (O). Here is the "Expected" table (E):

	White	Black	Total
Removed by Judge	1822.1	480.9	2303
Removed by Prosecutor	1734.2	457.8	2192
Removed by Defense	2569.7	678.3	3248
Total	6126	1617	7743

   The table of $\frac{(O-E)^2}{E}$ looks like this:

4.8	18.0
50.9	193.0
59.3	224.6

The sum of those values is 550.6 (with slight adjustments for rounding error). The most extreme value in the chi-square table with $df = 2$ is 13.82 which corresponds to a p-value of 0.001. Obviously, this is way beyond that. [The actual p-value based on the χ^2_2 distribution is about 10^{-120} but I wasn't expecting you to calculate that! If you took thw calculation far enough to conclude that it's "off the charts," that's good enough for me.]

(c) The overall retention rate is higher for whites than blacks but not by very much (less than 2% different and, while it's statistically significant at 5%, it wouldn't be at the more stringent 1% standard). However there's a *much* bigger racial discrepancy in the rates of disqualifications by judges, prosecutors and defence attorneys, with judges and prosecutors disqualifying a higher percentage of black jurors and the defense disqualifying a higher percentage of white jurors. The main thing I hoped you would pick up was this sharp distinction between the overall bias in jury selection, which is relatively mild, and the much stronger bias that occurs within each of the three categories of challenges by judges, prosecutors or defenders. [The prosecution always get to make jury challenges before the defense, and the authors of the study suggest that the defense often try to compensate by making challenges to white jurors in about the same proportions as the prosecution make challenges to blacks; thus, there is some balancing but not enough to eliminate the racial bias altogether.]

7. (a) $b_1 = \frac{R \times s_y}{s_x} = -\frac{0.331 \times 3.423}{21.51} = -0.05267$ and $b_0 = \bar{y} - b_1\bar{x} = 3.703 + 0.05267 \times 37.5 = 5.6781$ to four decimal places.

(b) See the figure at the end of this answer sheet. The easiest way to get this is to compute the y values associated with $x = 1$ and $x = 74$ (extreme ends of the plot): these come to $5.6781 - 0.05267 \times 1 = 5.625$ and $5.6781 - 0.05267 \times 75 = 1.778$; mark these points in the graph (large red dots), then draw a straight line from one to the other.

(c) $5.6781 - 0.05267 \times 75 = 1.73$, $5.6781 - 0.05267 \times 97 = 0.569$. (Small departures from these answers due to rounding error will not be penalized.)

(d) The straight line would cross 0 when $x = -\frac{b_0}{b_1} = 107.8$ or, rounding up, $x = 108$ which corresponds to the year 2051.

(e) Obviously, a prediction of < 0 days below 32 degrees in a year does not make sense, so there must be something wrong with the assumptions. The most critical one is that the response (y) variable is not normally distributed — it must be an integer and cannot go below 0. Another apparently faulty assumption is constant variability — the variability seems to be decreasing with time. As an alternative method, we could do it by logistic regression if we recoded each individual day (from December 1 to December 31 each year) as 1 if the temperature was below 32 and 0 otherwise; then the response is a binary variable. (But one of the assumptions of logistic regression, independent observations, would not be satisfied in this case, since obviously there is some dependence from one day to the next. Therefore, this is not an ideal solution either.)

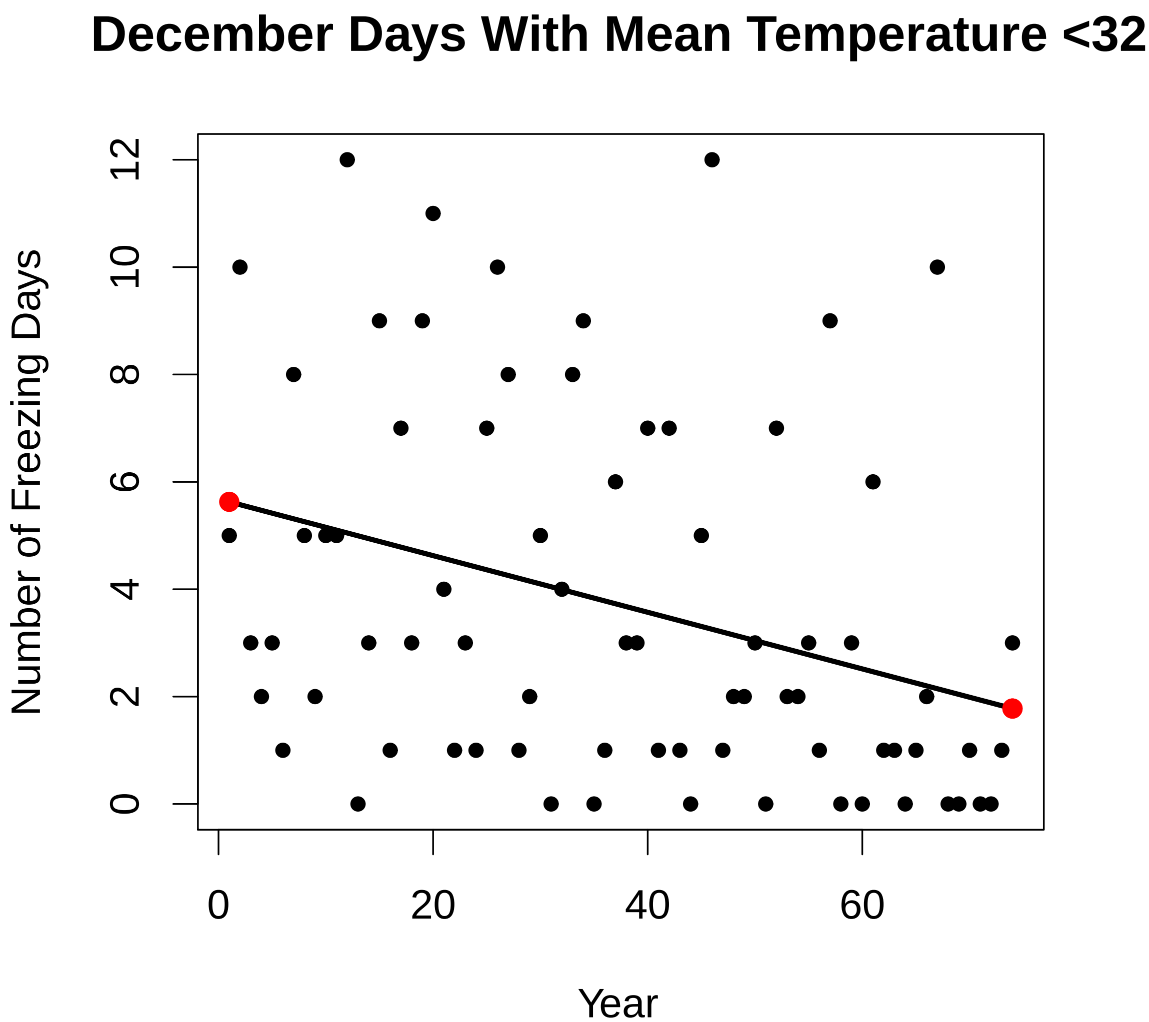
8. (a) $\hat{p}_1 = \frac{2685}{24179} = 0.111$ with a standard error $\sqrt{\frac{0.111 \times (1-0.111)}{24179}} = 0.00202$. The 95% confidence interval is $0.111 \pm 1.96 \times 0.00202 = (0.107, 0.115)$ to three decimal places.

(b) Let p_1 be the proportion of non-returned absentee ballots in Anson County and let p_2 be the proportion of non-returned absentee ballots in Richmond County. To three decimal places, $\hat{p}_1 = 0.036$ and $\hat{p}_w = 0.029$ (from the table). Under the null hypothesis $H_0: \; p_1 = p_2$, the pooled estimate of p_1 and p_2 is $1 - \frac{3857+6987}{4000+7197} = 0.0315$ so the standard error of $\hat{p}_1 - \hat{p}_2$ is $SE = \sqrt{0.0315 \times (1 - 0.0315) \times \left(\frac{1}{4000} + \frac{1}{7197}\right)} = 0.00344$, and the z statistic for the difference is $\frac{\hat{p}_1 - \hat{p}_2}{SE} = 2.03$. Because $2.03 > 1.96$, the result is statistically significant as a two-sided test at the 0.05 level of significance. [In fact, a more precise calculation retaining more significant digits in $\hat{p}_1 = 0.03575$ and $\hat{p}_2 = 0.02918$ leads to $z = 1.91$ which is not statistically significant at the 5% level. I didn't realize when setting the question that the answer depends so critically on the level of rounding but I would accept either answer so long as the method of calculation was clear.]

(c) Given your answers to parts (a) and (b) it is hardly necessary to repeat the full calculation for (c), since the result is very obviously statistically significant, but the full calculation leads to: $\hat{p}_1 = 0.111, \hat{p}_2 = 0.0245$. The pooled value of $\hat{p}$ is 0.03878 which leads to a standard error $\sqrt{0.03838 \times (1 - 0.03878) \times \left(\frac{1}{24179} + \frac{1}{122281}\right)} = 0.00136$ and a z statistic of $\frac{0.111-00245}{0.00136} = 63.6$ which is of course extremely significant.

(d) Write X for the number of missing absentee votes that would have gone to McCready if they had been counted. If we assume a binomial distribution with $n = 2100$ and $p = 0.6$ then the mean of X is $2100 \times 0.6 = 1260$ and the standard deviation $\sqrt{2100 \times 0.6 \times (1 - 0.6)} = 22.45$.

(e) There are several possible answers here and I would accept any of them if carefully argued. If you require only that McCready pick up 905 votes then from (d) the answer is yes — it seems virtually certain that McCready had more than 905 votes from the 2100 that went missing. However, if you require that McCready pick up 905 votes *more than his opponent* (ignoring the third-party candidate), that leads to McCready needing at least 1503 votes among the 2100 to be counted, and by (d), that does not seems likely. However, there is a third argument that doesn't use probabilities as all — if Republican party workers went through the votes and removed all of them that voted for McCready, then an argument could be made that the whole 2100 votes belonged to McCready! [Since I originally set this question, new and quite different allegations have emerged and even Republicans are now acknowledging that there will have to be a revote.]


December Days With Mean Temperature <32
Number of Freezing Days
0
2
4
6
8
10
12
0
20
40
60
Year