

STOR 155: SPRING 2024
Midterm One, February 8, 2024

YOUR NAME (PRINT).....

PID.....

Please sign the **pledge**: "On my honor, I have neither given nor received unauthorized aid on this examination."

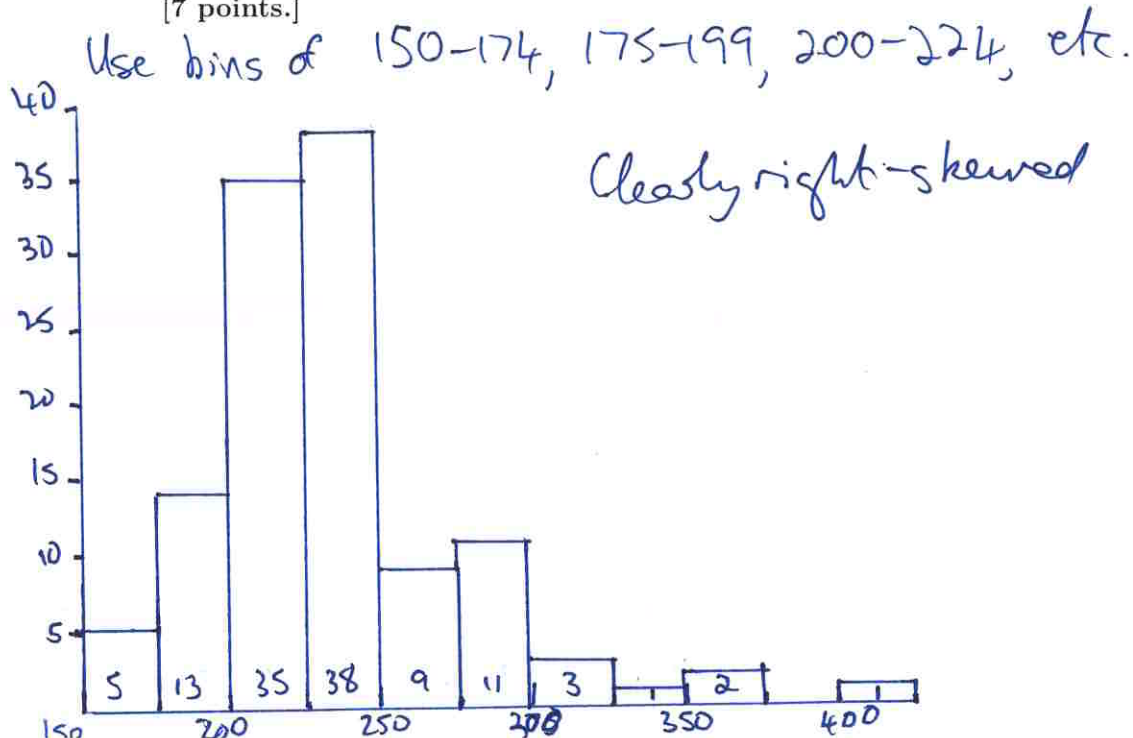
Sign.....

Open book in-class exam: time limit 75 minutes. Numbers in **boldface** indicate the number of points per part-question (100 points total for the whole exam).

1. A fan of the singer Taylor Swift has compiled a list of the lengths of 118 songs recorded by Ms. Swift. Converted to seconds and arranged in decreasing order, the list is as follows:

403 367 352 327 317 311 302 298 295 294 293 293 290 287 285 283 281 275
 271 267 265 263 261 261 260 254 250 248 247 **245** 245 244 244 243 242 242
 242 242 240 240 240 239 238 238 237 237 237 237 236 236 235 235 235
 234 234 234 233 **232 232** 231 231 231 230 230 230 230 227 225 223 222 222
 221 220 220 220 220 219 217 217 217 215 214 213 213 212 211 211 **211** 211
 209 208 207 207 207 204 203 203 203 201 201 201 200 200 199 198 195 193
 193 191 190 190 178 173 173 171 170 150

- (a) Draw a histogram of this distribution, taking care to choose suitable location and width of the bins. Would you say the distribution is right skewed, left skewed or symmetric? [7 points.]

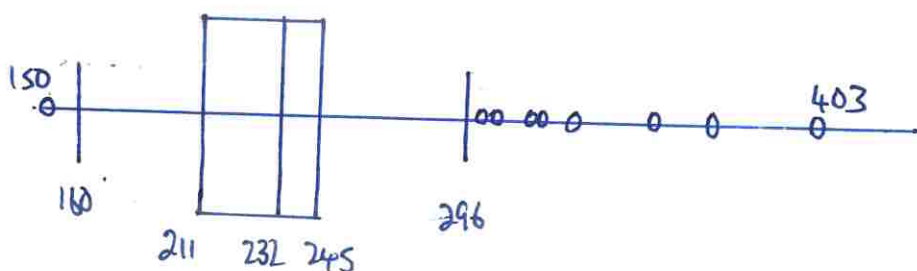


- (b) What are the median, first quartile, third quartile and IQR of this distribution? [7 points.]

232, 211, 245, 34

- (c) Draw a boxplot, taking care to note the positions of the outer whiskers and identifying outliers. [7 points.]

$$Q3 + 1.5 \times IQR = 245 + 51 = 296 \quad Q1 - 1.5 \times IQR = 211 - 51 = 160$$



- (d) Do you think the mean of this distribution is greater than or less than the median? Briefly state why. [5 points.]

Greater than, because the distribution is right-skewed.

- (e) Here is a comment made by a reader of the website where this was posted: "Someone told me (and I have no idea how credible they are) that songs are getting shorter because of streaming. There's apparently some statistic about spotify users on average only listening to about 2 and a half minutes of a song, and if a song isn't played all the way through, it doesn't count as a play. So artists in general, not just Taylor, have started releasing shorter songs in order to get the most number of plays for a song."

Based on your statistical analysis, would you say the data support this conclusion? Note that the data only contain the lengths of the songs and not their date of release. [7 points.]

Various points you could make here. One is that without knowing the dates of the songs, you can't tell whether they are getting shorter over time. Another point is that all but one of the songs is over $2\frac{1}{2}$ minutes, so in a literal sense the claim is false. But you could also point out that the distribution is clearly not normal and there seem to be a lot of songs in the $2\frac{1}{2}$ -4 minute range.

2. A survey of Americans' responses to the DREAM Act (the proposed legislation to grant a path to citizenship for illegal immigrants who were brought to the USA as children) produced responses as follows. Participants were asked their political ideology (classified as Conservative, Liberal or Moderate) and whether they supported the Act.

Ideology	Support for DREAM Act			Total
	No	Not sure	Yes	
Conservative	151	35	186	372
Liberal	52	9	114	175
Moderate	161	28	174	363
Total	364	72	474	910

One person is chosen at random from the respondents to this survey.

- (a) What is the probability that the selected respondent is a Liberal? [5 points.]

$$\frac{175}{910} = 0.192$$

- (b) What is the probability that the selected respondent supports the DREAM Act? [5 points.]

$$\frac{474}{910} = 0.521$$

- (c) What is the probability that the selected respondent is a Liberal and supports the DREAM Act? [5 points.]

$$\frac{114}{910} = 0.125$$

- (d) Are the events "selected respondent is a Liberal" and "selected respondent supports the DREAM Act" independent? Explain why or why not. [6 points.]

$$0.192 \times 0.521 = 0.100 \neq 0.125$$

Therefore, not independent.

- (e) Given that the selected respondent is opposed to the Dream Act, what is the probability that he/she is a Conservative? [6 points.]

$$\frac{151}{364} = 0.415$$

- (f) What is the probability that the selected respondent is not sure about the DREAM Act or is not a Moderate? [6 points.]

If $A =$ "not sure about DREAM", $B =$ "not a Moderate"

$$P(A) = \frac{72}{910} = 0.079 \quad P(B) = \frac{547}{910} = 0.601 \quad P(A \text{ and } B) = \frac{44}{910} = 0.048$$

$$\text{so } P(A \text{ or } B) = 0.079 + 0.601 - 0.048 = 0.632$$

3. A certain type of cancer can be in one of two states: benign or malignant. 1.5% of Americans have this cancer, and among those, 20% are malignant. A diagnostic test is proposed. If the person being tested has a malignant cancer, the probability that the test will be positive is 91%. If it's a benign cancer, the probability of a positive result is 85%. If the person has no cancer, there's still an 8% chance of a false positive result. (Assume that the result of the test is either positive or negative: further tests such as a biopsy would be needed to distinguish between benign and malignant).

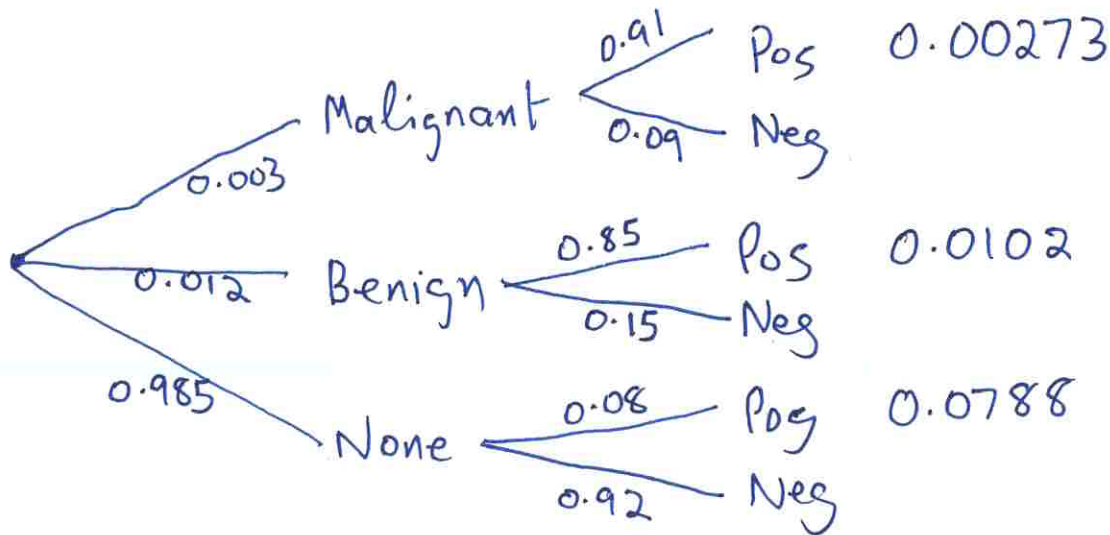
- (a) What is the unconditional probability that the person selected for testing has (i) the cancer in its malignant form?, (ii) the cancer in its benign form?

[7 points.]

$$(i) 0.015 \times 0.2 = 0.003 \quad (ii) 0.015 \times 0.8 = 0.012$$

- (b) Given that the test comes back positive, what is then the probability that the person being tested has (i) the cancer in its malignant form?, (ii) the cancer in its benign form?

[27 points.]



$$P\{\text{positive test result}\} = 0.00273 + 0.0102 + 0.0788 = 0.09173$$

$$P\{\text{Malignant} | \text{Pos Test}\} = \frac{0.00273}{0.09173} = 0.0298 \quad \text{answer to (i)}$$

$$P\{\text{Benign} | \text{Pos Test}\} = \frac{0.0102}{0.09173} = 0.111 \quad \text{answer to (ii)}$$