STOR 151, Spring 2024, Homework 7

Reading: course text up to Section 4.3 and the "Handout on Hypergeometric Distribution" (course webpage).

Problems for solution:

1. Exercise 4.18, page 156 of course text
2. Exercise 4.20, page 156 of course text
3. Exercise 4.24, page 157 of course text
4. Suppose we take a regular deck of cards and discard everything except the hearts. There are 13 cards, three of which are "picture cards" (jack, queen, king), the rest of the cards being labelled ace, two, three and so on up to 10. The 13 cards are turned over and three of them drawn at random.

(a) What is the probability that all three cards are picture cards?
(b) What is the probability that one of them is a picture card and two of them are not?
(c) Now suppose the rules are changed so that we draw cards one at a time, and keep drawing until the first picture card. What is the probability that this occurs on the fifth drawing?

Hint for Problem 4. This uses the hypergeometric distribution; see the handout on the course webpage. Also, I'd like to clarify that the cards are drawn *without replacement* (under this drawing scheme, you cannot get the same card twice.)

Lengthy answers are not required, but you should be sure to address each part of the question in your answer.

Hand in on gradescope. If uploading as a single pdf file, please "assign" your pages.

Due time and date: 11:59 pm Friday, March 8, 2024

Solution (note: I am writing (n k) for the "choose" function n!/(k!(n-k)!))

(a) (10 0)x(3 3)/(13 3)=1/(13 3). But (13 3)=13x12x11/6=286 so the answer is 1/286=0.0035 (R notation: dhyper(3,3,10,3))
(b) (10 2)x(3 1)/(13 3)=45x3/286=0.4720 (dhyper(1,3,10,3))
(c) The chance of no picture cards on the first four drawings is (10 4)x(3 0)/(13 4)=210x1/715=0.2937 (dhyper(0,3,10,4)). After that, the chance of a picture card on the fifth drawing is 3/9=1/3, so the final answer is 0.2937/3=0.0979.