STOR 151 SECTION 2 MIDTERM 2
MARCH 25 2010

This is an open book exam. Course text, personal notes and calculator are permitted. You have 75 minutes to complete the test. Personal computers and cellphones are not to be used during the exam. If you have any queries about the meaning of the questions, or if you think there is an error, ask the proctor for assistance. Answers are to be written in a blue book.

Before you begin:

(a) Write your name and PID on the front of your blue book.

(b) Sign the "pledge". If this is not preprinted, copy out the following statement and sign it: *On my honor, I have neither given nor received unauthorized aid in this exam.*

SHOW ALL WORKING — even correct answers will not get full credit if it's not clear how they were obtained. Incorrect answers will gain substantial credit if the method of working is substantially correct.

Answer all questions. The score for each part is indicated at the end of the question (total 100).

1. A random sample of voters are asked which political party they support (Republican, Democrat or Independent) and whether they support the Health Care Bill (yes or no). Those who did not answer, or gave some answer other than those, were not included in the survey.

   Among those who responded, 45% said they were Republican, 35% said they were Democrat, and 20% said they were Independent. Among the Republicans, 10% supported the Bill; among Democrats, 70% supported the Bill; among Independents, 55% supported the Bill.

   (a) Represent this information in the form of a tree diagram. [**10**]

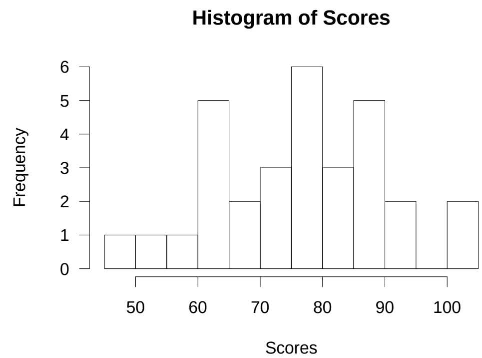
   (b) What proportion of all the voters were Independents who opposed the Health Care Bill? [**7**]

   (c) What proportion of all the voters supported the Health Care Bill? [**7**]

   (d) Given that a voter supported the Health Care Bill, what is the probability that he or she is (i) a Republican, (ii) a Democrat, (iii) an Independent? [**10**]

[**CONTINUED ON OTHER SIDE. PLEASE TURN PAGE.**]

2. During the 2009–2010 season just concluded, the UNC women's basketball team played 31 games. The points scored per game had a mean of 76.5, a standard deviation of 13.7, and a distribution quite close to a normal distribution (see inset histogram). For the remainder of this question, assume that the number of points scored in a single game actually does follow a normal distribution with this mean and standard deviation.



(a) State an interval of scores within which the number of points scored in a single game almost certainly lies. [**6**]

(b) Based on the normal distribution, what is the probability that the team scores more than 90 points in a single game? [**7**]

(c) Based on the normal distribution, what is the probability that the team scores between 60 and 75 points in a single game? [**7**]

(d) Based on the normal distribution, what are (i) the first quartile, (ii) the third quartile and (iii) the interquartile range of the distribution of point scores? [**7**]

(e) The first quartile, third quartile and inter-quartile range of the actual scores are 65.5, 88 and 22.5 respectively. Comparing with your answers to (d), what does this tell you about how well the normal distribution actually fits the data? [**6**]

3. My four-year-old son likes M&Ms (for anyone who doesn't know — a type of candy) but he especially likes the ones that are red. In a large batch of M&Ms, approximately 32% are red. Assume the different colors are all mixed up at random in any given pack of M&Ms.

For parts (a)–(d), assume he counts out six M&Ms. We are interested in knowing how many of these are red.

(a) What probability distribution is appropriate in this situation? Explain the assumptions that you are making here. [**5**]

(b) What is the probability that none of the six M&Ms is red? [**5**]

(c) What is the probability that exactly two are red? [**6**]

(d) What is the probability that at least two are red? [**6**]

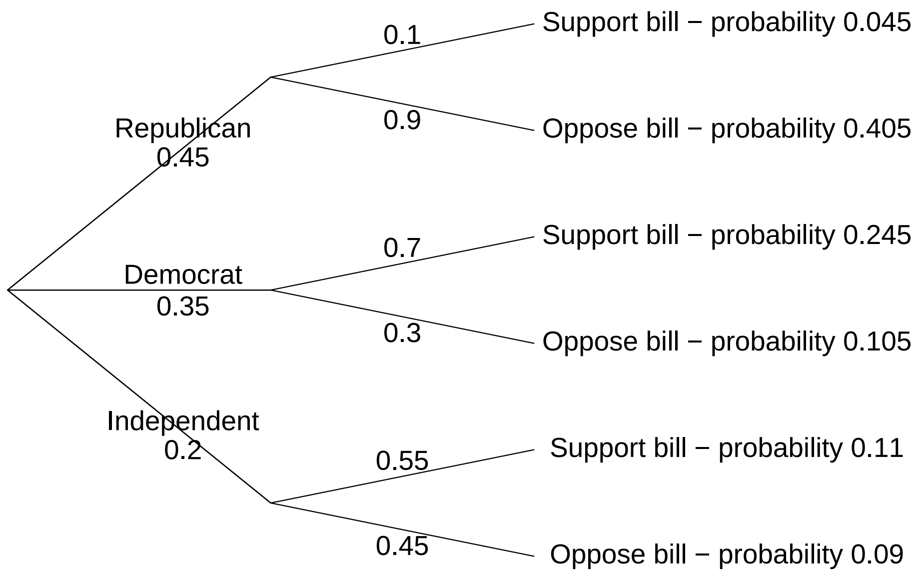
Now consider a large pack that contains 110 M&Ms.

(e) What probability distribution is appropriate in this situation? Explain why. [**5**]

(f) What is the probability that at least 45 of the M&Ms in the box are red? [**6**]

SKETCH SOLUTIONS

1. **(a)** See below.



   (b) .09 or 9%

   (c) .045+.245+.11=0.4 or 40%

   (d) (i) $\frac{.045}{.4} = .1125$. (ii) $\frac{.245}{.4} = .6125$. (iii) $\frac{.11}{.4} = .275$. (Answers that have been rounded off or expressed as percentages are also acceptable.)

2. **(a)** $76.5 \pm 3 \times 13.7 = 76.5 \pm 41.1 = (35.4, 117.6)$.

   (b) $z = \frac{90-76.5}{13.7} = 0.99$ for which the left-hand probability from Table A is 0.8389. So the probability of scoring more than 90 is $1 - 0.8389 = 0.1611$ or 16% approximately.

   (c) For $x = 60$, $z = \frac{60-76.5}{13.7} = -1.20$, probability 0.1151. For $x = 75$, $z = \frac{75-76.5}{13.7} = -0.11$, probability 0.4562. Difference $0.4562 - 0.1151 = 0.3411$, or 34% approximately.

   (d) The z scores associated with the first and third quartiles are –0.67 and +0.67 (from Table A or the homework assignment just completed). The corresponding x scores are therefore $76.5 - 0.67 \times 13.7 = 67.32$ and $76.5 + 0.67 \times 13.7 = 85.68$. Since basketball scores have to be whole numbers, I would quote these as (i) 67 and (ii) 86. The IQR is then (iii) 86–67=19. (Note: since none of these numbers is exact, I will accept small variations on these calculation.)

   (e) The actual first quartile is slightly lower, and the third quartile slightly higher, than the values calculated from the normal distribution; the IQR is slightly higher. This implies that the normal distribution slightly underestimates the true variability of the basketball scores. There is no evidence that the distribution is skewed left or right.

3. **(a)** Binomial distribution with $n = 6$, $p = 0.32$. Assumptions (i) independent, (ii) same probability applies to each M&M.

(b) $(1 - 0.32)^6 = 0.0989$ or about 10%.

(c) $\frac{6!}{4!2!} \times 0.32^2 \times 0.68^4 = 15 \times .1024 \times .2138 = .3284$ or about 33%.

(d) Compute the complementary probability. The probability that exactly one is red is $6 \times 0.32 \times 0.68^5 = .2792$, so the required answer is $1 - .2792 - .0989 = 0.6219$ or about 62%.

(e) The normal distribution can be applied because $np = 35.2$, $n(1-p) = 74.8$, both much bigger than 15.

(f) The mean is $\mu = np = 35.2$ and the standard deviation is $\sigma = \sqrt{np(1-p)} = \sqrt{110 \times 0.32 \times 0.68} = 4.892$. Corresponding to $x = 45$, we have $z = \frac{45-35.2}{4.892} = 2.00$, for which the left-hand tail probability is 0.9772. Therefore, the answer is $1 - 0.9772 = .0228$, or about 2.3%.