STOR 151 SECTION 2 FINAL EXAM
MAY 1 2010

YOUR NAME:

PID:

Honor pledge: On my honor, I have neither given nor received unauthorized aid in this exam.

SIGNATURE:

Please write your answers in a blue book, except for the graph in Question 2 which is at the end of this exam. **Please hand in this question paper as well as your blue book.**

This is an open book exam. Course text, personal notes and calculator are permitted. You have 3 hours to complete the test. Personal computers and cellphones are not allowed. If you have any queries about the meaning of a question, ask the instructor for advice.

SHOW ALL WORKING — even correct answers will not get full credit if it's not clear how they were obtained. Incorrect answers will gain substantial credit if the method of working is substantially correct.

Answer six of the eight questions. If you attempt more than six, all the answers will be graded but only the best six (complete questions) will count. Each question is worth a total of 20 points and the whole exam is worth 120 points (which will be rescaled to a maximum of 35 for grading purposes). Points for each individual part of a question are also given in square brackets.

1. Two independent throws are made of an 8-sided die (exactly like a regular die, but there are 8 faces instead of 6 — each of the numbers 1 through 8 is equally likely to come up). The total of the two throws is noted, which could be any number between 2 and 16.

   Consider the events

   A: the first throw is one of 2, 3, 6 or 7

   B: the first throw is odd

   C: the sum of the two throws is 13 or greater

   (a) Are the events A and B independent? [**6**]
   (b) Are the events A and C independent? [**7**]
   (c) Are the events B and C independent? [**7**]

2. A professor of English believes that students who watch an excessive amount of TV score badly in a reading comprehension test. She administers the test to 10 students and records their scores (y). She also asks them how many hours of TV they watch per week (x). The results are:

Student	1	2	3	4	5	6	7	8	9	10
TV Hours x	0	1	8	6	8	3	1	20	2	1
Score on Test y	64	59	53	78	60	58	71	20	63	39

You can assume the following without checking: $\bar{x} = 5,\ \bar{y} = 56.5,\ s_x = 6.1,\ s_y = 16.5,\ r = -0.65$.

(a) Find the coefficients, a and b, in the regression line $\hat{y} = a + bx$. [**7**]

(b) A scatterplot of the x and y values is shown at the back of this question paper. Draw in the regression line from part (a) on that scatterplot. [**5**]

(c) What is the predicted score for a student who watches 40 hours of TV per week? Does this seem a reasonable estimate? — explain why or why not. [**5**]

(d) Comment on the regression analysis. Are there any unusual features that might make you wary of the conclusion? [**3**]

3. A casino game is played in the following way. The player has to pay \$10 to enter the game. Then, a fair coin is tossed three times. If all three come up heads, the player collects a prize of \$50. When that happens, the coin is tossed a further seven times. If all seven of those also come up heads (so there have been ten heads in a row), the player receives an *additional* \$2,000.

(a) Write down a table of the possible payouts and their respective probabilities. [**5**]

(b) What is the mean payout to the player on a single play of the game? [**5**]

(c) An observer writes down the actual payout for each play over a long sequence of plays. The standard deviation of these payouts is calculated to be about \$66. Based on this information, if $\bar{x}$ denotes the *mean* payout after n plays of the game, what are the mean and standard deviation of $\bar{x}$? [**3**]

(d) The manager of the casino reckons that the game is played 900 times in the course of a single evening. What is the probability that by the end of the evening, the casino has made a profit? [**7**]

4. A medical research organization would like to conduct a survey to estimate the proportion of people with diabetes in a native American tribe. A random sample of tribe members will be selected and tested for diabetes. The researcher would like to construct a confidence interval for the proportion of tribe members who have diabetes, with confidence coefficient 99% and a margin of error no more than .028.

(a) What sample size is needed if the researcher makes no prior assumption about the true rate of diabetes in the population? [**6**]

(b) It is known that the overall proportion of people with diabetes in the American population is about .08. Based on this information, the researcher is willing to assume that the rate of diabetes in the native American tribe is not more than .15. If the sample size is recalculated under this assumption, what sample size is then needed? [**6**]

(c) In fact, the test proceeds with a sample of size 921, among whom 166 are found to have diabetes. Find a 99% confidence interval for the true proportion of tribe members who have diabetes. Based on this confidence interval, do you think the assumption the researcher made in (b) was justified? [**8**]

5. A brand of beer advertises that it contains 5% alcohol. In order to test that claim, a consumer organization collects 5 bottles of beer and measures their alcohol content. The results for the 5 bottles are: 4.2%, 4.6%, 4.6%, 4.4%, 4.2%.

   (a) Assume the 5 bottles of beer are a random sample from the population of all bottles of beer of this particular brand. Construct a 95% confidence interval for the mean alcohol content of the brand. [**8**]

   (b) Does this test provide statistically significant evidence that the true mean alcohol content in the beer is less than 5%? Summarize the evidence in terms of a P-value (as closely as you are able to determine it), and explain how you interpret the conclusion. [**8**]

   (c) What are the assumptions underyling this statistical procedure? Do you believe they are satisfied in this example? [**4**]

6. A recent New York Times/CBS News Poll interviewed 881 adults who identified themselves as supporters of the Tea Party, and a separate sample of 1580 from the general population. Among the Tea Party supporters, 41% stated that they believed President Obama was born in the United States. Among the general population sample, 58% stated that they believed President Obama was born in the United States.

   (a) Find a 90% confidence interval for the difference between the proportion of Tea Party supporters who believe President Obama was born in the United States and the proportion of the general population who believe President Obama was born in the United States. [**10**]

   (b) Do you believe the two proportions are equal? Formulate this as a hypothesis testing problem, carry out the test, and state your conclusions. [**10**]

7. A medical researcher believes that the diastolic blood pressure is on average higher among men than among women. To test this hypothesis, he takes a random sample of 6 men, and a random sample of 6 women, and tests them in pairs, one man and one woman being tested in the same room at the same time. The results are as follows:

Pair	1	2	3	4	5	6
Woman's diastolic blood pressure	86	78	80	70	66	82
Man's diastolic blood pressure	92	92	100	74	76	94

   The means of the women's and men's samples are 77 and 88, and the standard deviations are 7.56 and 10.51, respectively.

   (a) Analyze the data as a two-sample experiment. Is there a significant difference between the mean diastolic blood pressure of women and men? Be sure to state your assumptions and the individual steps of the test. [**8**]

   (b) Analyze the data as a matched pairs experiment. Is there a significant difference between the mean diastolic blood pressure of women and men? Be sure to state your assumptions and the individual steps of the test. [**8**]

   (c) Which of the two tests in (a) and (b) do you consider more appropriate on this situation? Based on that, summarize your overall conclusion. [**4**]

8. An opinion survey question asked "Do you think gun control laws should be made stricter, less strict, or kept the same as they are now?" The survey was originally conducted in 1994 but repeated in 2002 and 2010. The responses were as follows:

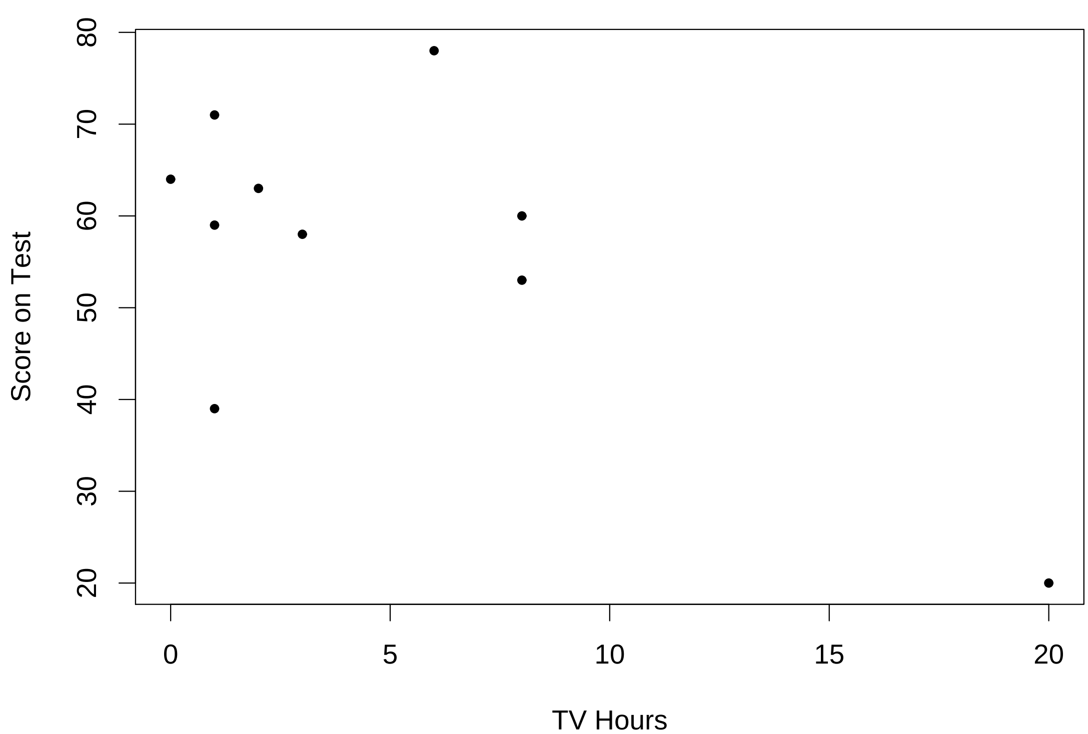
Year	Stricter	Less Strict	Kept the Same
1994	65	11	30
2002	75	14	40
2010	63	47	86

(a) Based on this survey, would you say that there is significant evidence that attitudes to gun control have changed over the time period between 1994 and 2002? Carry out a formal hypothesis test and state your conclusions. [**12**]

(b) Describe in your own words the pattern of the change, using calculations from part (a) to support your answer. [**3**]

(c) Suppose that, among all those individuals who favored stricter gun control in 2002, I want to estimate the proportion of that group who had changed their minds by 2010. Does the above table contain adequate information to calculate this proportion? Based on the information available, what would be your best estimate? [**5**]

For Question 2, please show your answer directly on the following graph.



SOLUTIONS

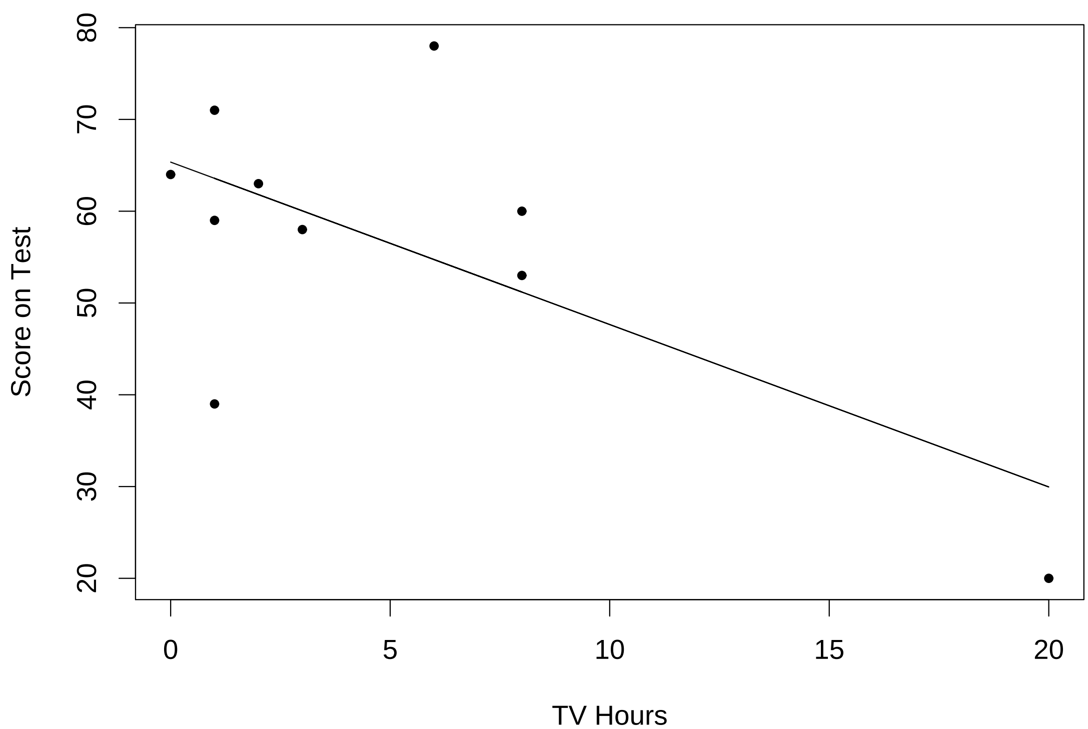
1. (a) The possible outcomes of the first die resulting in the event A are 2, 3, 6, 7; probability $\frac{4}{8} = \frac{1}{2}$. The possible outcomes of the first die resulting in the event B are 1, 3, 5, 7; probability $\frac{4}{8} = \frac{1}{2}$. The possible outcomes of the first die resulting in the event "A and B" are 3, 7; probability $\frac{2}{8} = \frac{1}{4}$. But $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, so the events A and B are independent.

   (b) The possible outcomes (first die, then second die) leading to the event C are (8,5), (7,6), (6,7), (5,8), (8,6), (7,7), (6,8), (8,7), (7,8), (8,8). That's 10 possibilities out of 64 in the sample space, so $P(C) = \frac{10}{64} = \frac{5}{32}$. The event "$A$ and C" contains the outcomes (7,6), (6,7), (7,7), (6,8), (7,8), so $P(A \text{ and C}) = \frac{5}{64}$. But $P(A) \times P(C) = \frac{1}{2} \times \frac{5}{32} = \frac{5}{64} = P(A \text{ and C})$. So A and C are independent.

   (c) The event "B and C" contains the outcomes (7,6), (5,8), (7,7), (7,8), so $P(B \text{ and C}) = \frac{4}{64} = \frac{1}{16}$. But $P(B) \times P(C) = \frac{1}{2} \times \frac{5}{32} = \frac{5}{64} \neq P(B \text{ and C})$. So B and C are not independent.

2. (a) $b = r\frac{s_y}{s_x} = -1.76, \; a = \bar{y} - b\bar{x} = 65.3$

   (b) See following figure:



   (c) $a + 40b = 65.3 - 40 \times 1.76 = -5.1$. This does not seem to be reasonable since the score is negative, but the problem is that we are extrapolating a long way beyond the range of the data.

   (d) Apart from the extrapolation in (c), it does appear that the observation for which $x = y = 20$ is an outlier — delete this one observation, and there does not appear to be any effect of TV watching on the exam score.

3. (a) See the following table for the payout x:

x	p	xp
0	7/8	0
50	127/1024	6.201
2050	1/1024	2.002

(b) The mean payout is \$(6.201+2.002)=\$8.203.

(c) The mean of $\bar{x}$ is 8.203 and the standard error is $\frac{66}{\sqrt{n}}$.

(d) Based on $n = 900$, the casino will make a profit if the mean payout per play of the game is less than \$10. For that probability, the z statistic is $\frac{10-8.203}{66/\sqrt{900}} = 0.82$. The associated left-tail probability is 0.7939. In other words, there is about an 80% chance the casino will make a profit over the course of the evening.

4. (a) The required sample size follows $n = p(1-p)\left(\frac{2.58}{0.028}\right)^2$, where p is the (assumed) true population proportion. When no value of p is given, the convention is to assume $p = 0.5$, which leads to $n = 2123$ (to the nearest whole number). The number 2.58 arises because this is the critical value of z associated with a two-sided tail probability of .05 (left-tail probability .975). This also comes from Table 8.2, page 369 of text.

(b) With $p = 0.15$, $n = 1083$.

(c) $\hat{p} = \frac{166}{921} = 0.1802$ so the 99% confidence interval is $\hat{p} \pm 2.58\sqrt{\frac{\hat{p}(1-\hat{p})}{921}} = 0.1802 \pm 0.0327 = (0.1475, 0.2129)$. The value .15 lies within this confidence interval, therefore the data are consistent with the prior assumption. However it is probably fair comment that this only came about because of the adoption of a 99% confidence coefficient — with a 90% or 95% confidence interval, this would not be true.

5. (a) The mean of the five samples is 4.4 and the standard deviation

x	$x - \bar{x}$	$(x - \bar{x})^2$
4.2	–0.2	0.04
4.6	0.2	0.04
4.6	0.2	0.04
4.4	0	0
4.2	–0.2	0.04
Total	0	0.16

is $\sqrt{\frac{0.16}{4}} = \sqrt{0.04} = 0.2$ (see table of calculations). The standard error is $\frac{0.2}{\sqrt{5}} = 0.0894$. The critical value of the t statistic for 95% confidence and $df = 4$ is 2.776 (from table), so the 95% confidence interval is $4.4 \pm 2.776 \times 0.0894 = (4.15, 4.65)$.

(b) The t statistic is $t = \frac{4.4-5}{.0894} = -6.711$. Ignoring the minus sign, 6.711 is between 4.604 and 7.173 and, therefore, the two-sided P-value is between 0.002 and 0.01 (from the table of the t distribution with $df = 4$). In this case, you are told to perform a one-sided test and, since H_a is that the mean strength of the beer is *less than* 5%, a negative value of t supports that hypothesis, with a one-sided P-value between 0.001 and 0.005. The interpretation is that since the P-value is mush smaller than .05, the data strongly support H_a, i.e. the mean alcohol content really is less than 5%.

(c) The assumptions are than the data are a random sample from a normal distribution with the same mean and standard deviation. There is no reason to think that these assumptions are not valid in this instance.

6. (a) $\hat{p}_1 = .58,\ \hat{p}_2 = .41$ based on sample sizes 1580 and 881 respectively. The standard errors are $\sqrt{\frac{.58\times.42}{1580}} = .0124$ and $\sqrt{\frac{.41\times.59}{881}} = .0166$, so the combined standard error for the difference is $\sqrt{.0124^2 + .0166^2} = .0207$. The 90% confidence interval for the difference is $.17 \pm 1.645 \times .0207 = (.136, .204)$.

   (b) For a hypothesis test, we compute the pooled $\hat{p} = \frac{1580\times.58+881\times.41}{1580+881} = .5191$ with a standard error for the difference of $\sqrt{.5191 \times .4809 \times \left(\frac{1}{1580} + \frac{1}{881}\right)} = .021$. The z statistic is $\frac{.17}{.021} = 8.095$ which is beyond the range of the normal probability table, so the (one-sided or two-sided) P-value is well less than .0001. The conclusion is that the difference between the tea party supporters and the general population is very clearly significant.

7. (a) The standard error for the standard two-sample test is $\sqrt{\frac{7.56^2+10.51^2}{6}} = 5.285$ so the t statistic is $\frac{11}{5.285} = 2.08$. Based on the t table with $df = 5$, the two-sided P-value is between .05 and .1. Not significant.

   (b) If treated as a matched pairs experiment, we consider the differences 6, 14, 20, 4, 10, 12 which have mean 11 and standard deviation 5.76 (values of $x - \bar{x}$ are –5, 3, 9, –7, –1, 1; values of $(x - \bar{x})^2$ are 25, 9, 81, 49, 1, 1; $s = \sqrt{\frac{25+9+81+49+1+1}{5}}$). The standard error is $\frac{5.76}{\sqrt{6}} = 2.35$ and the t statistic is $\frac{11}{2.35} = 4.68$. The 2-sided P-value (from the t table with $df = 5$) is between .002 and .01, so it clearly *is* statistically significant. The assumptions are that the individual differences form a random sample from a normal distribution with unknown mean and variance.

   (c) Since the pairing of men and women was random, there is no reason to treat them as matched. Therefore, the first interpretation of the experiment is more appropriate. Based on this, the overall conclusion is that there is no statistically significant difference between the men and women. However, one could qualify that by noting that the sample size is very small, and a larger sample could well produce a statistically significant result.

8. (a) Set it up as a chi-squared test. The expected values (reading across the rows) are 49.93, 17.71, 38.37, 60.76, 21.55, 46.69, 92.32, 32.74, 70.94. Values of $\frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$ are 4.55, 2.54, 1.82, 3.34, 2.65, 0.96, 9.31, 6.21, 3.20; total 34.6. With df=4, this is clearly beyond the range of the chi-squared table (actual P-value is about .0000005).

   (b) The largest values of $\frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$ are in the third row, first two columns — the strongest direction of change is that the proportion of people supporting stricter gun control in the most recent survey is substantially down from previous surveys. Overall, there is a trend towards people favoring less gun control.

   (c) If we let A be the event that a randomly selected individual supported gun control in 2002, and B the event that the same individual supports gun control in 2010, then the survey suggests $P(A) = \frac{75}{129} = .581$ and $P(B) = \frac{63}{196} = .321$. The probability we are trying to calculate is $P(B^c \mid A) = \frac{P(B^c \text{ and } A)}{P(A)}$. We cannot estimate that, because the survey did not contain information to enable us to estimate joint probablities (in other words, we can estimate $P(A)$ and $P(B)$, but not $P(A \text{ and } B)$). It would be acceptable to end there. However, if we assume $P(A^c \text{ and } B) = 0$ (this is equivalent to saying that nobody who is in favor of gun control now was against it in 2002), then we would conclude that $P(B^c \text{ and } A) = P(A) - P(B \text{ and } A) = P(A) - P(B) = .260$. In that case $(B^c \mid A) = \frac{.260}{.581} = .45$.

General Comments on the Exam

The following table may be informative:

Question	1	2	3	4	5	6	7	8
Number Attempting	32	96	35	94	95	95	82	53
Mean Score	6.1	17.3	9.4	18.0	14.8	17.1	14.9	14.9

Question 1 was the least well answered of the whole exam. A lot of students just wrote yes or no (to each part) without any explanation. I decided to give 2 points for a correct answer, 0 for an incorrect answer, all the other points were for the derivation. A full answer required listing all the possible outcomes leading to each of the events A, B, C, A and B, A and C, B and C. Very few students did that.

Question 2 was generally well answered but a lot of students didn't mention extrapolation as the reason for a meaningless answer in (c).

Question 3 was also rather poorly answered — a lot of students misinterpreted part (a) but I gave nearly full credit if you wrote down a slightly different table for x and p and then did the rest of the question correctly.

Question 4 was generally well answered.

Question 5 was generally well answered, though some students interpreted it as a question about sample proportions, which I think is why the overall score was lower than on questions 2, 4 and 6.

Question 6 was generally well answered.

Question 7 was generally well answered. In part (a), some students assumed it was a one-sided test and then concluded that the result was statistically significant. So long as you made clear that you were treating it as one-sided, I gave credit for that. In part (b), a lot of students stated $s = 2.95$ apparently because $2.95 = 10.51 - 7.56$. However there is no such rule!

In **Question 8** there was an unfortunate ambiguity in the wording of the question — in (a), I meant to say the time period between 1994 and 2010 (not 2002). A lot of students just performed the chi-squared test on the first two rows, in which case the chi-square statistic is 0.25 which is definitely not statistically significant. Given that this was a defensible interpretation of the question as worded, I gave full credit for that. [A side comment is that it casts light on what was actually happening — the change in attitudes seems to have occurred between 2002 and 2010, not between 1994 and 2002 — this question was based on a real survey, though I changed the numbers to make them more amenable to the chi-squared calculations.]